COV886 Special Module in Algorithms: Computational Social Choice

Lecture 7

Incentives in the Stable Matching Problem

Feb 26, 2022 | Rohit Vaish

Reminder about starting recording

Stable Matching Problem

$w_1 > w_2 > w_3$

$m_3 > m_2 > m_1$

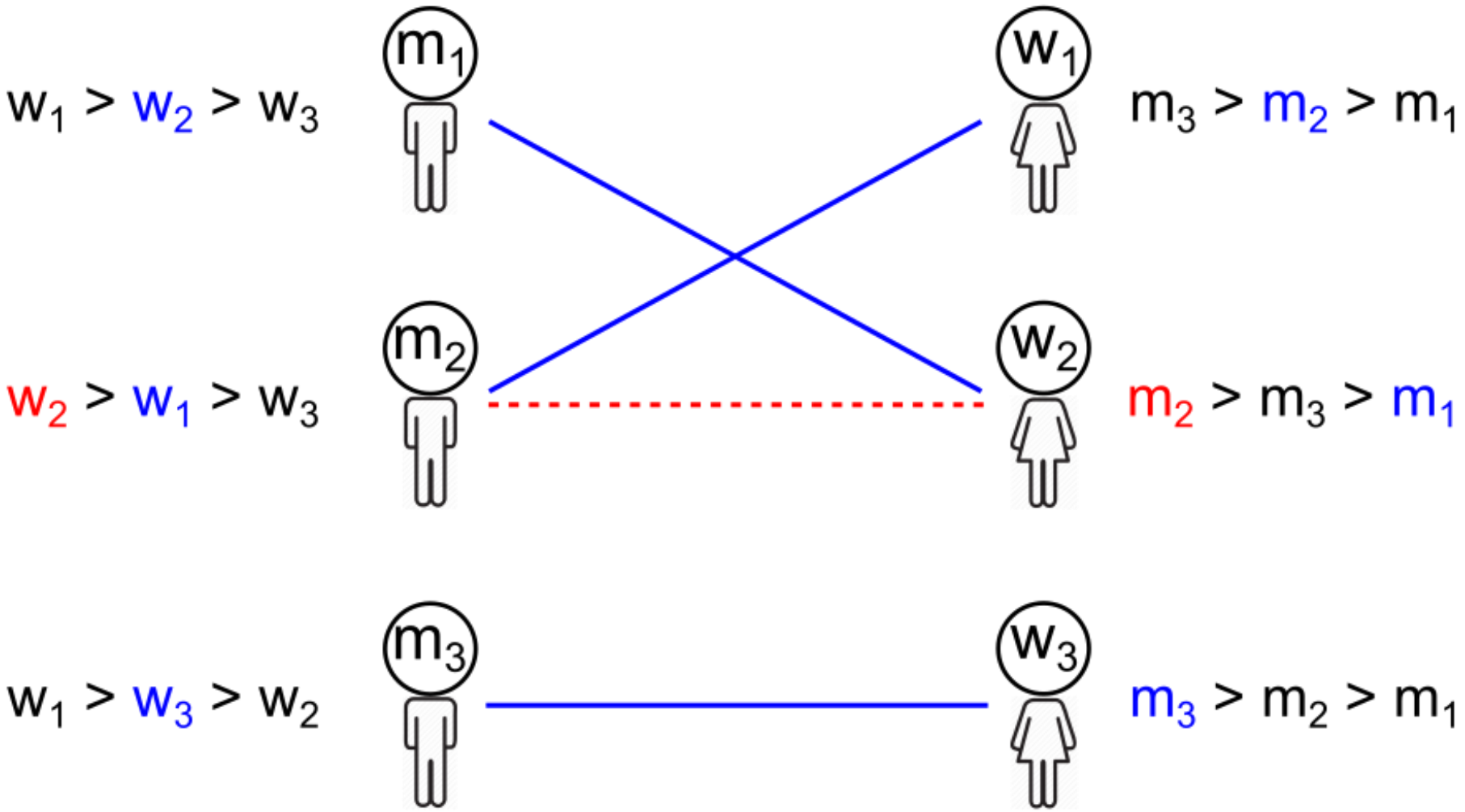
$w_2 > w_1 > w_3$

$m_2 > m_3 > m_1$

$w_1 > w_3 > w_2$

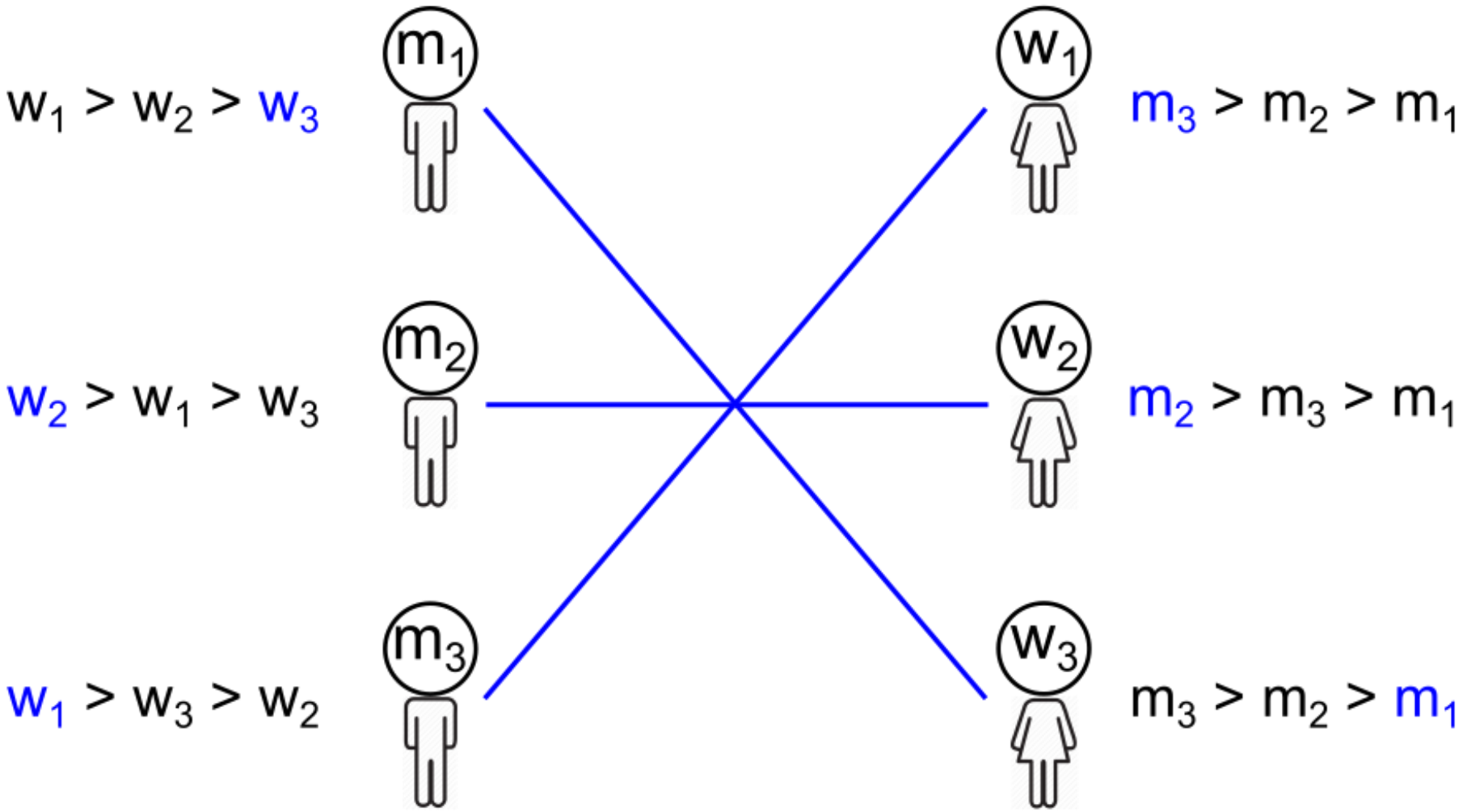
$m_3 > m_2 > m_1$

Stable Matching Problem



A matching is stable if there is no blocking pair.

Stable Matching Problem



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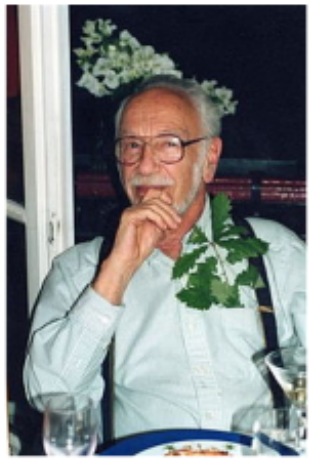
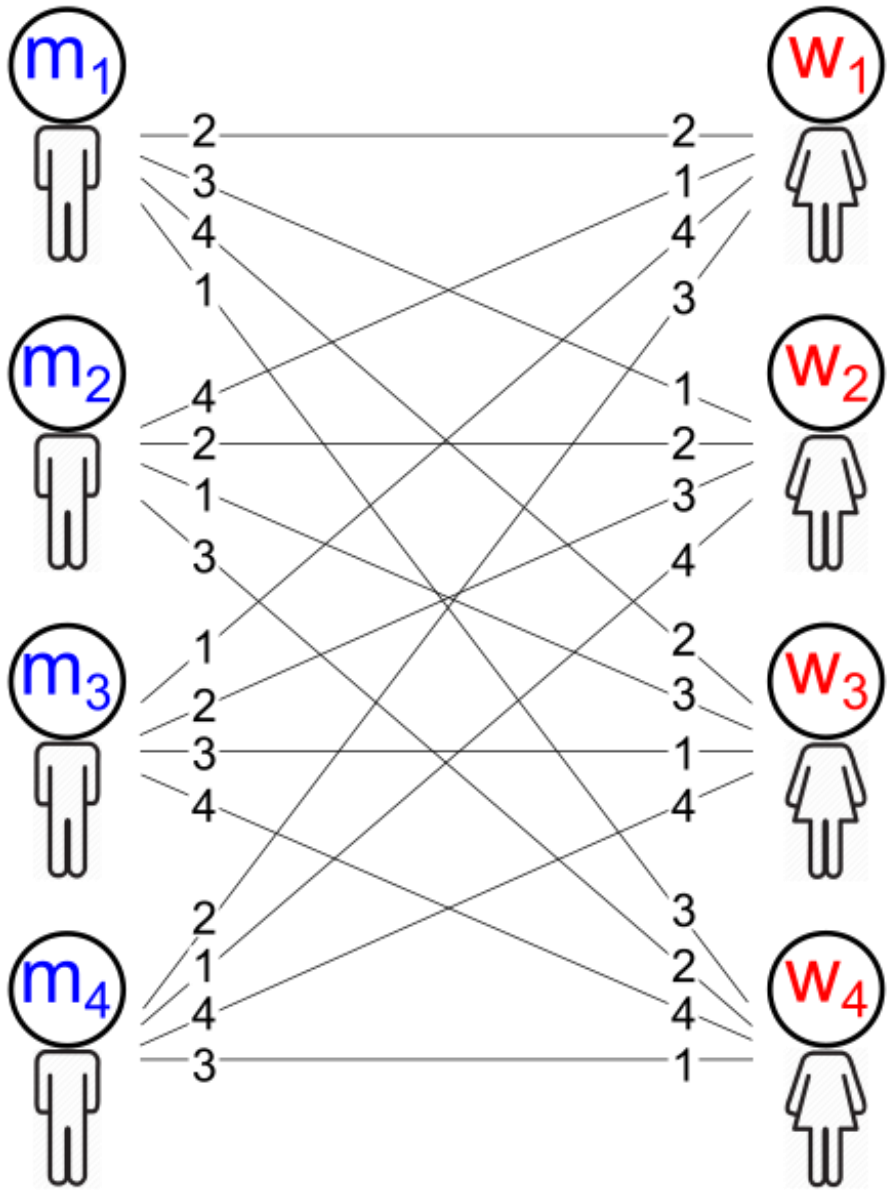


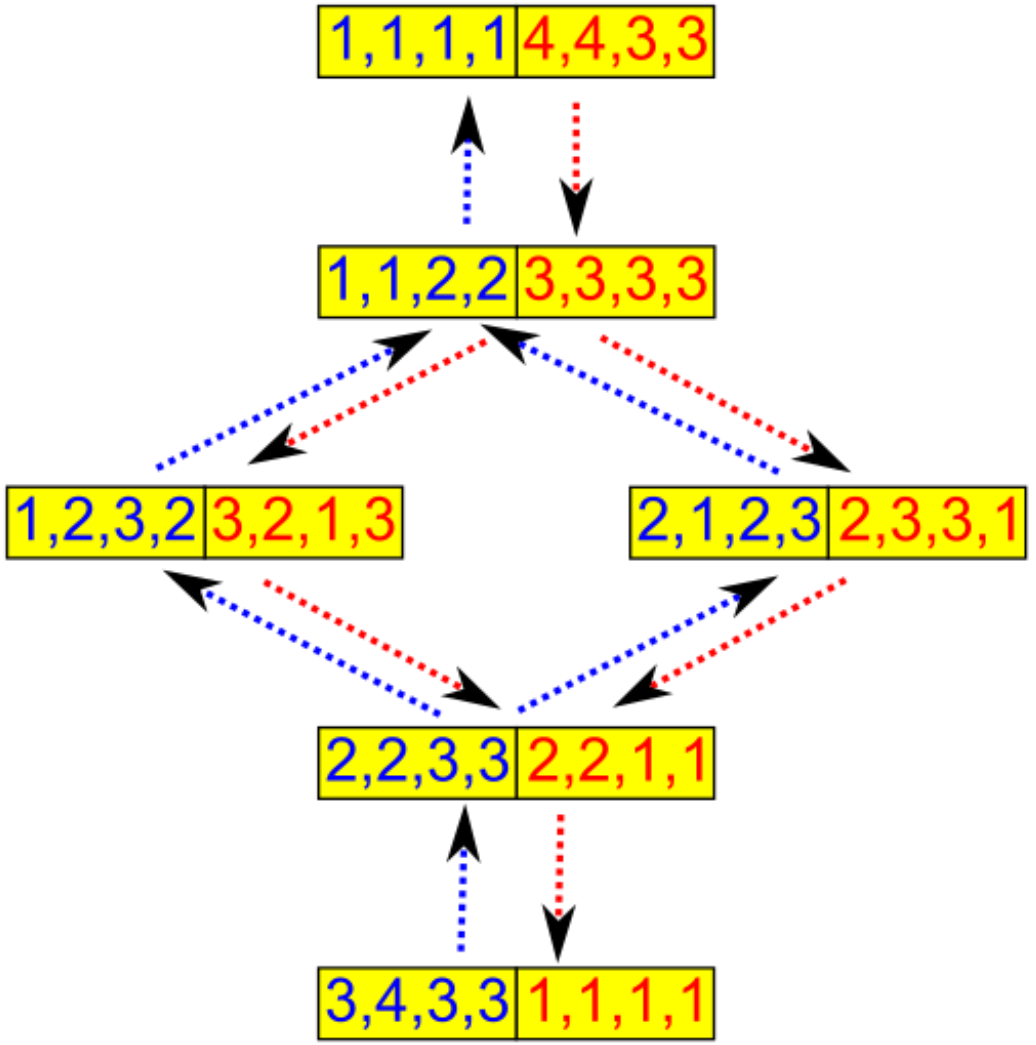
COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

Source: *The American Mathematical Monthly*, Jan., 1962, Vol. 69, No. 1 (Jan., 1962), pp. 9-15

Given any preference profile, a stable matching for that profile always exists and can be computed in polynomial time.


m1
m2
m3
m4
w1
w2
w3
w4
2 3 4 1
4 2 1 3
1 2 3 4
2 1 4 3
2 1 4 3
1 2 3 4
2 3 1 4
3 2 4 1


1,1,1,1 4,4,3,3
1,1,2,2 3,3,3,3
1,2,3,2 3,2,1,3
2,1,2,3 2,3,3,1
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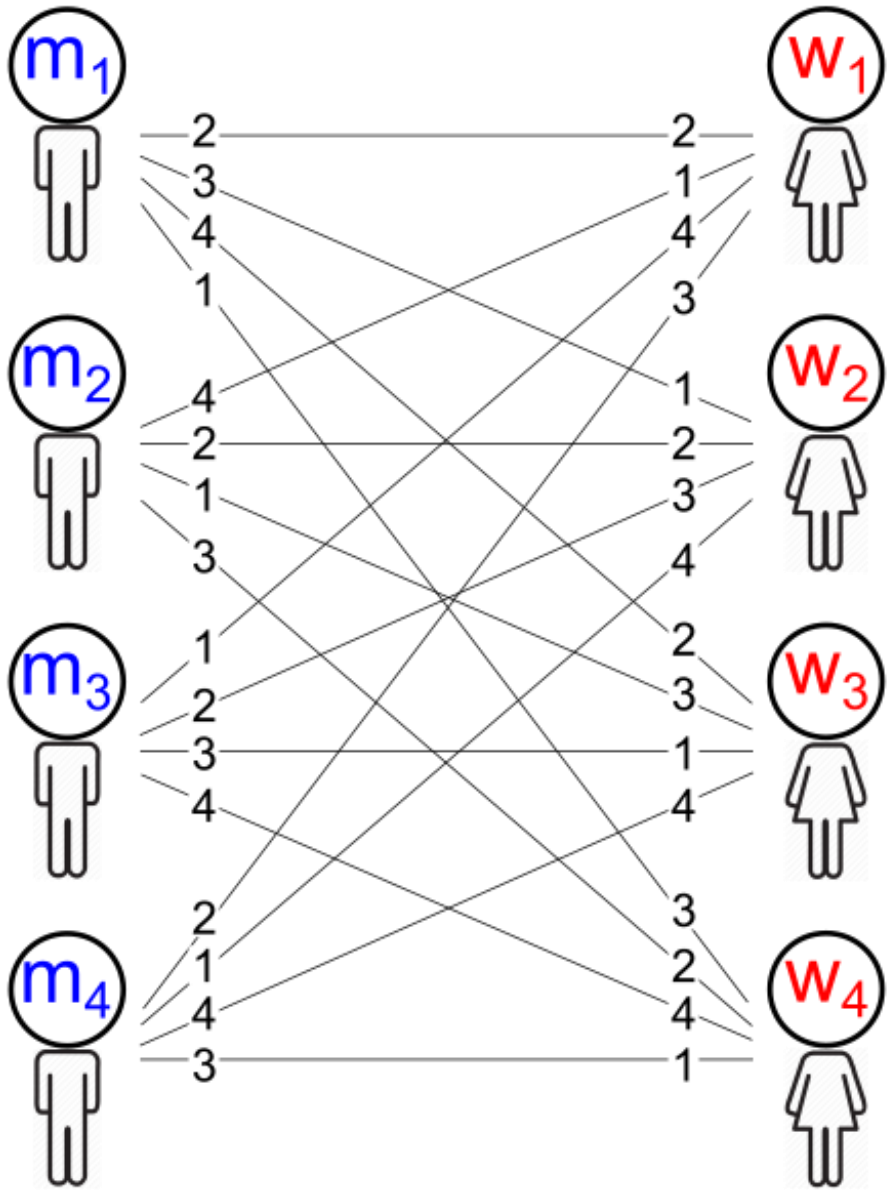
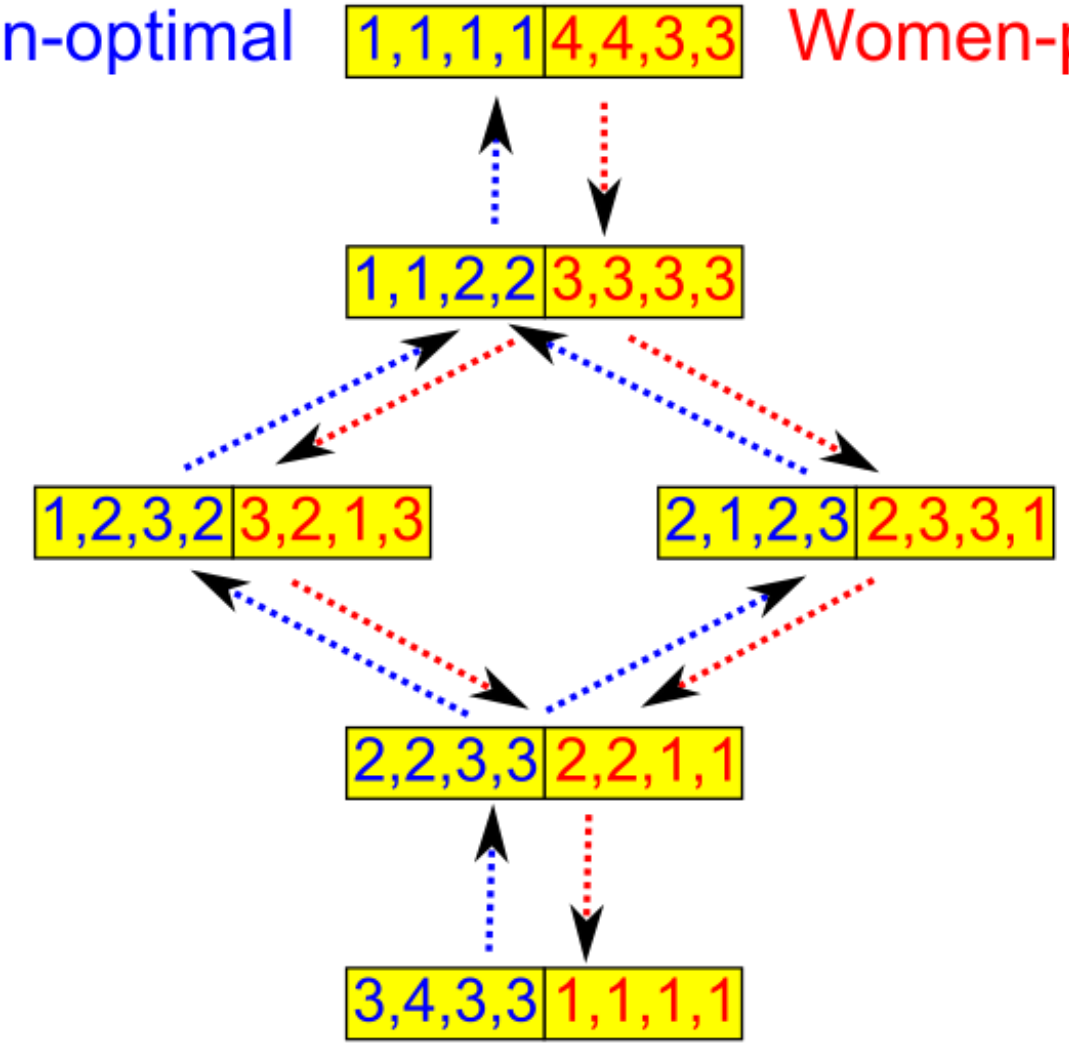
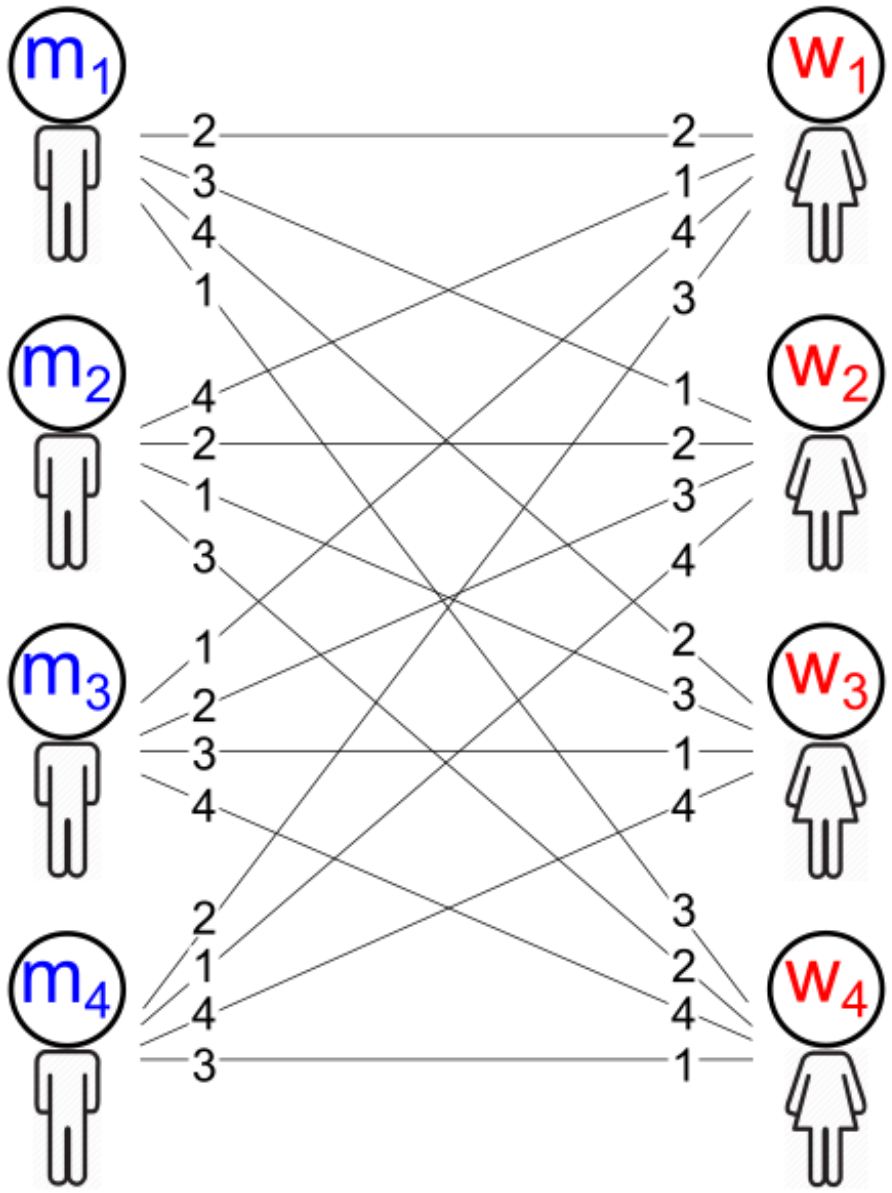


Men-optimal

Women-pessimal

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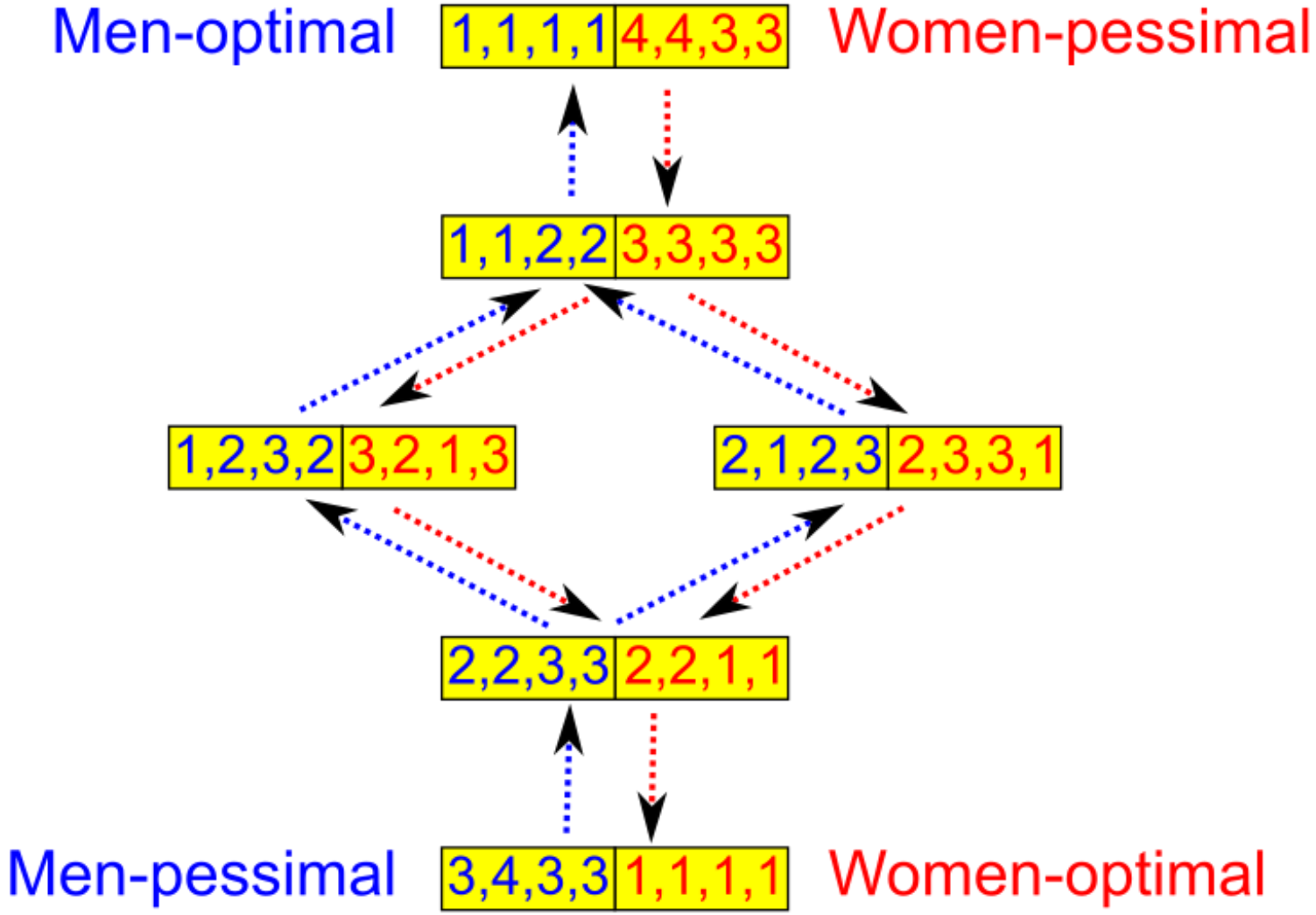
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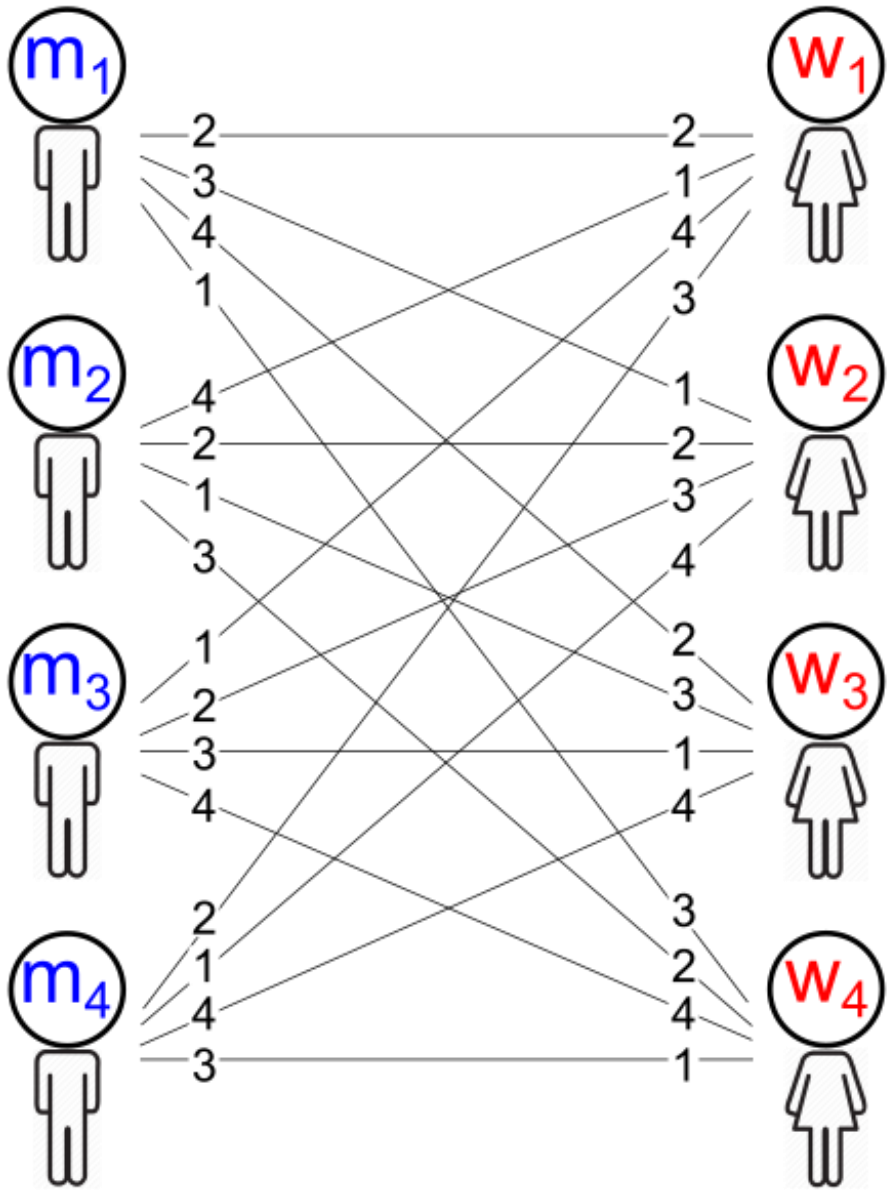
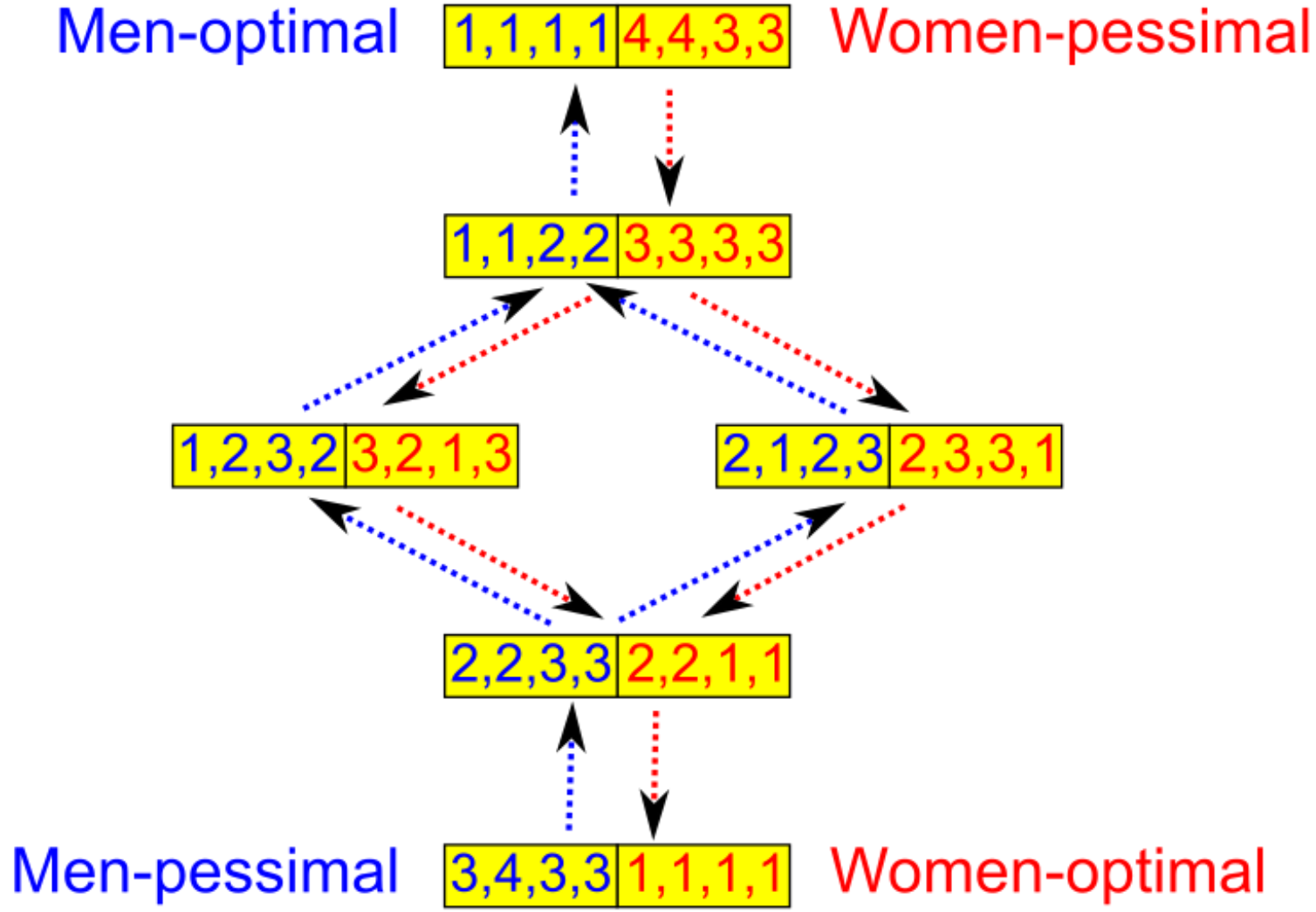
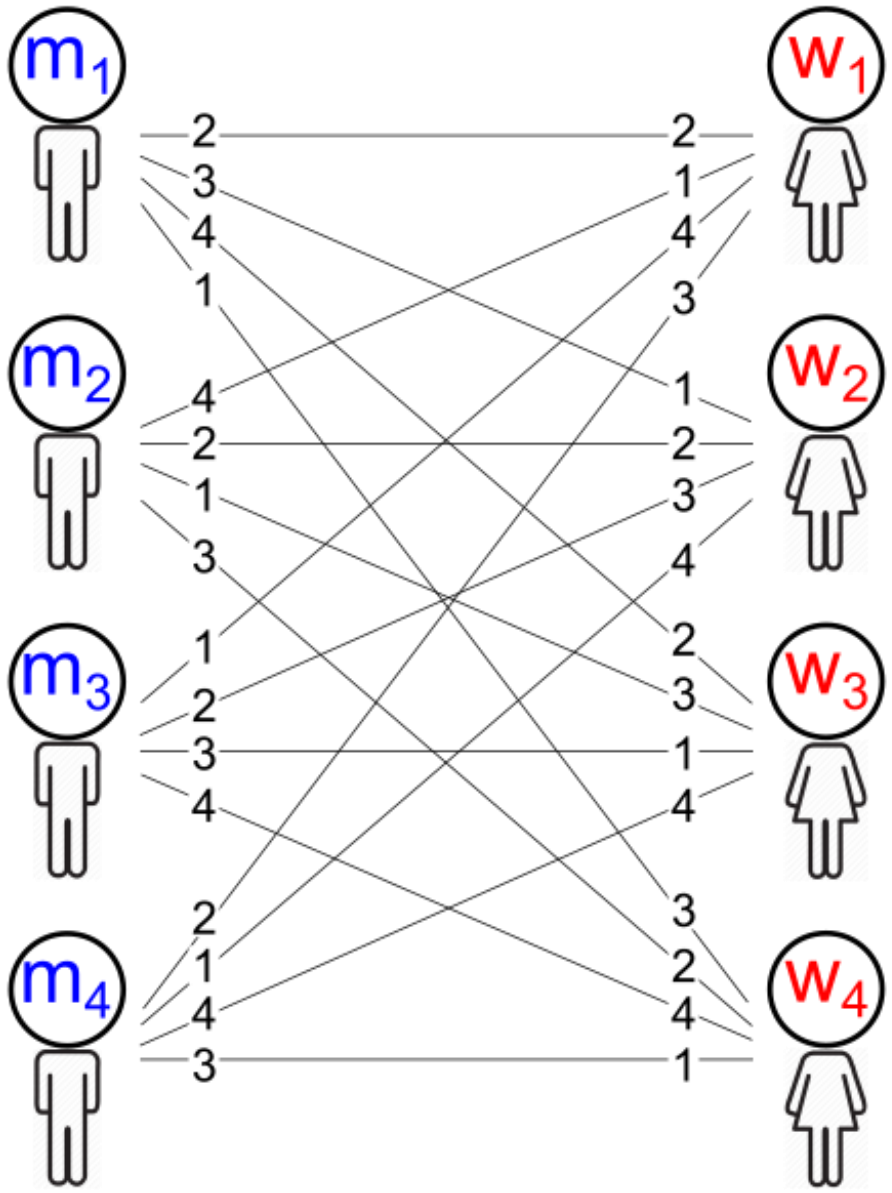


Men-proposing DA algorithm computes this





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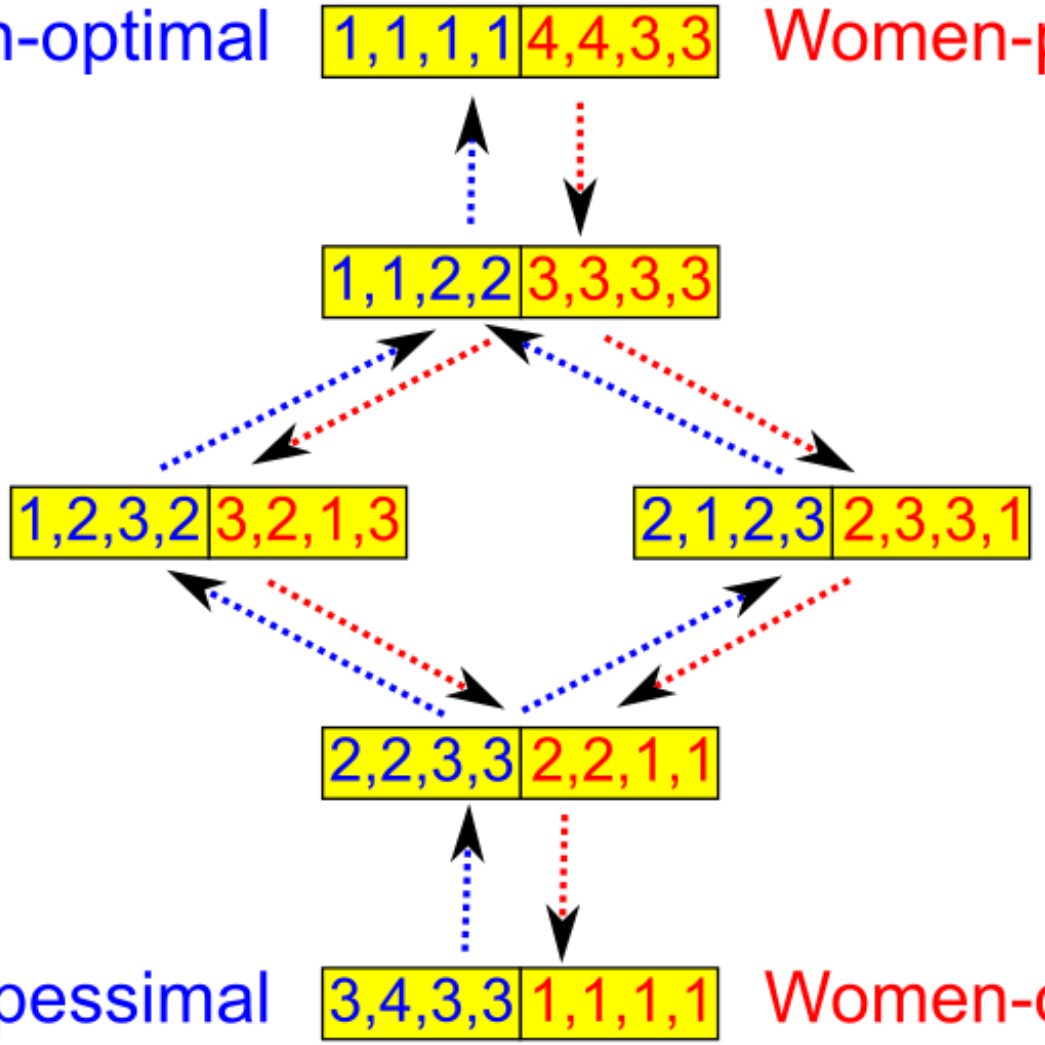

Women-pessimal

Men-pessimal

Women-optimal

Women-proposing DA algorithm computes this

DA algorithm can prevent blocking pairs.

Does it incentivize agents to report their preferences truthfully?

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is not strategyproof.

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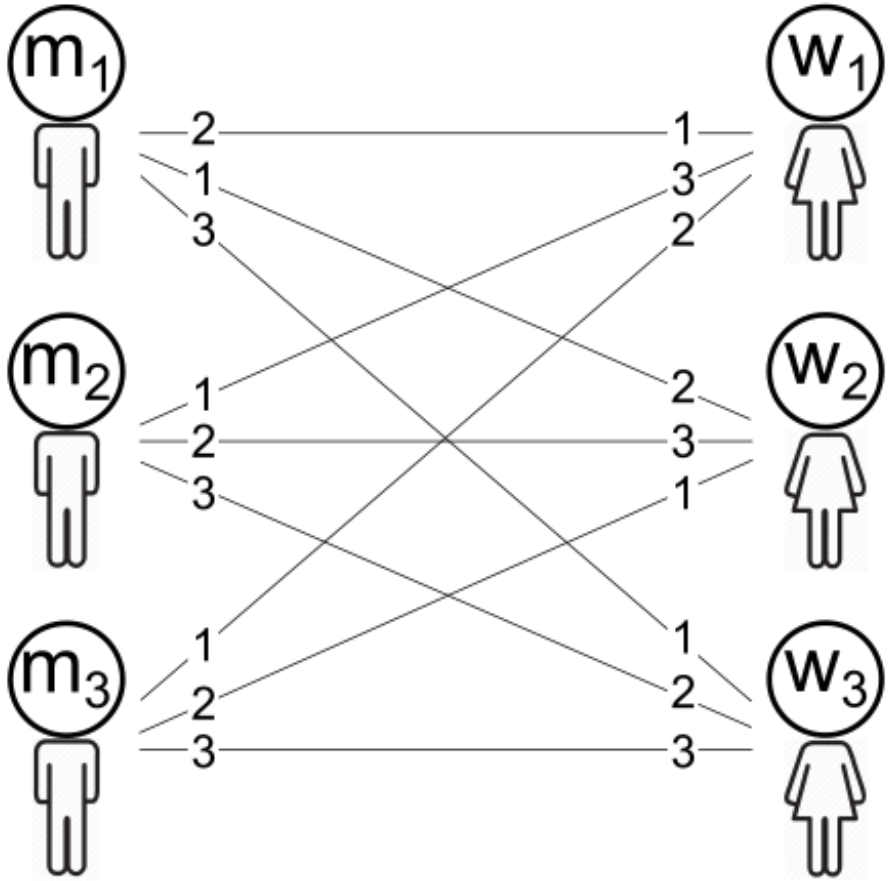

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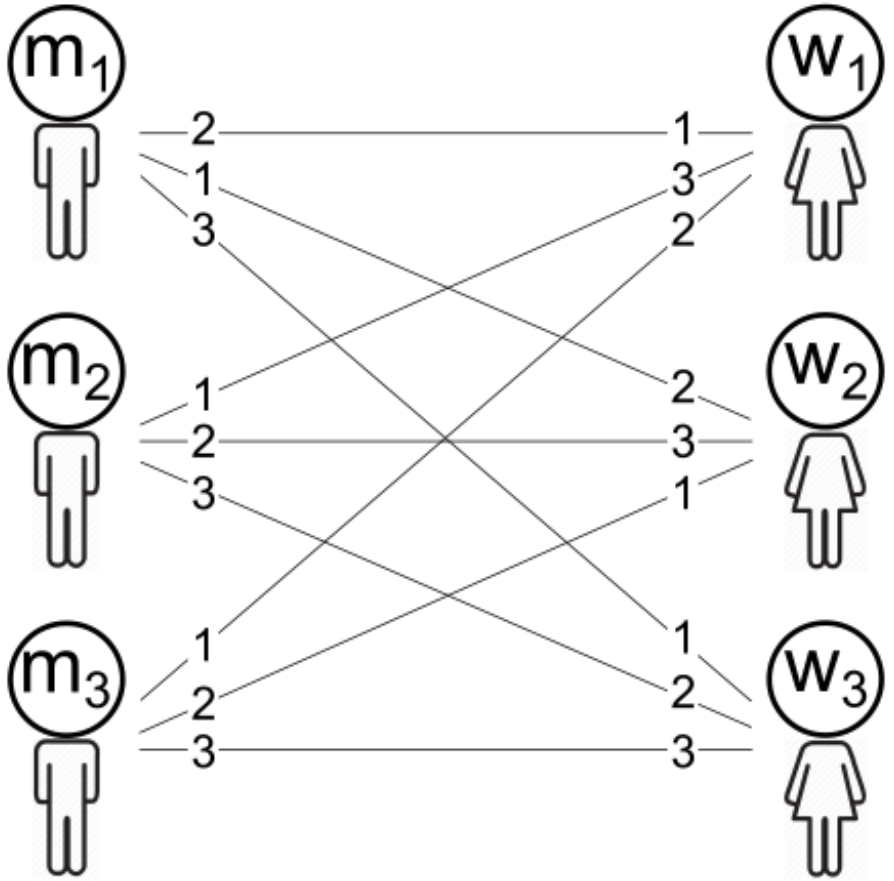

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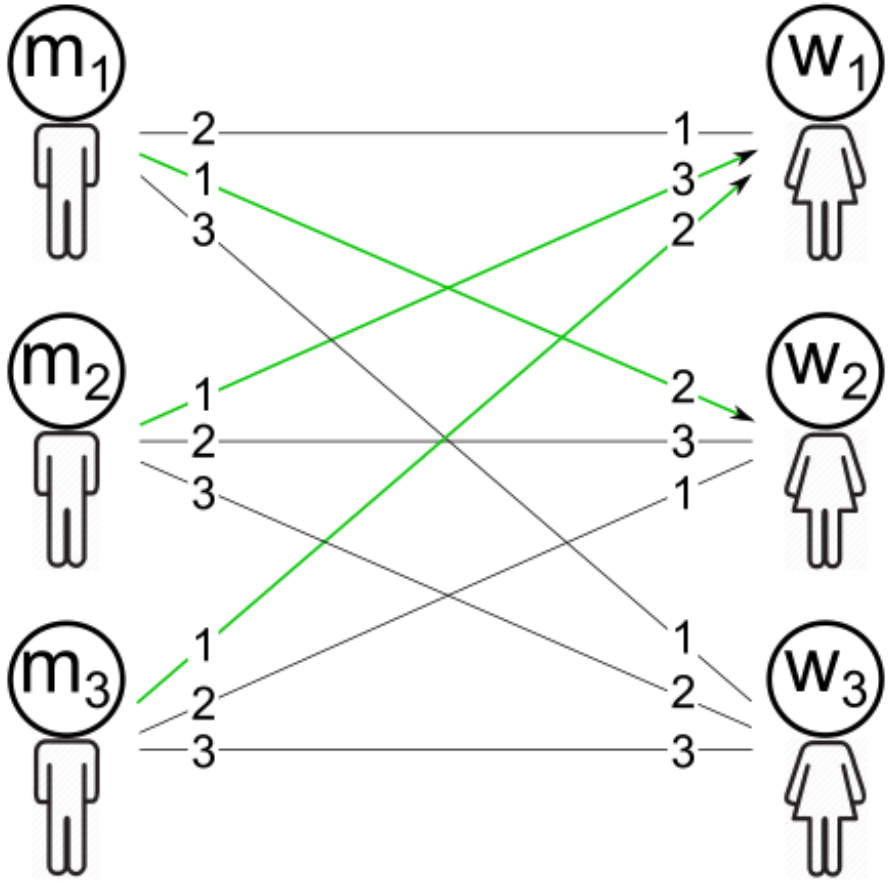

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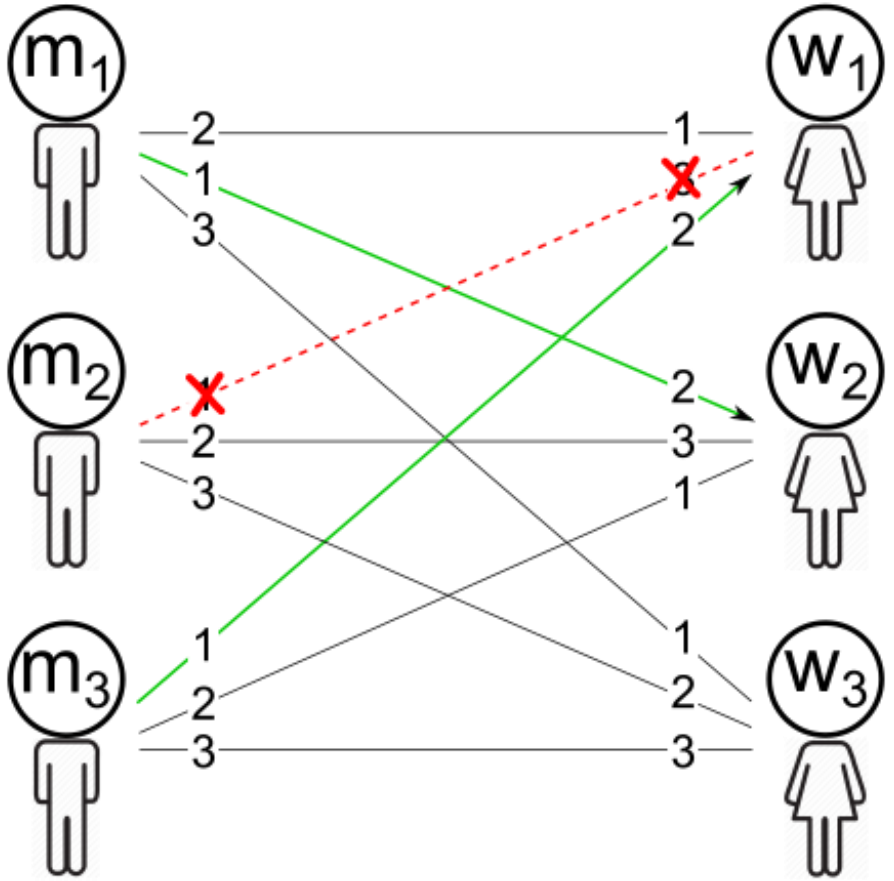

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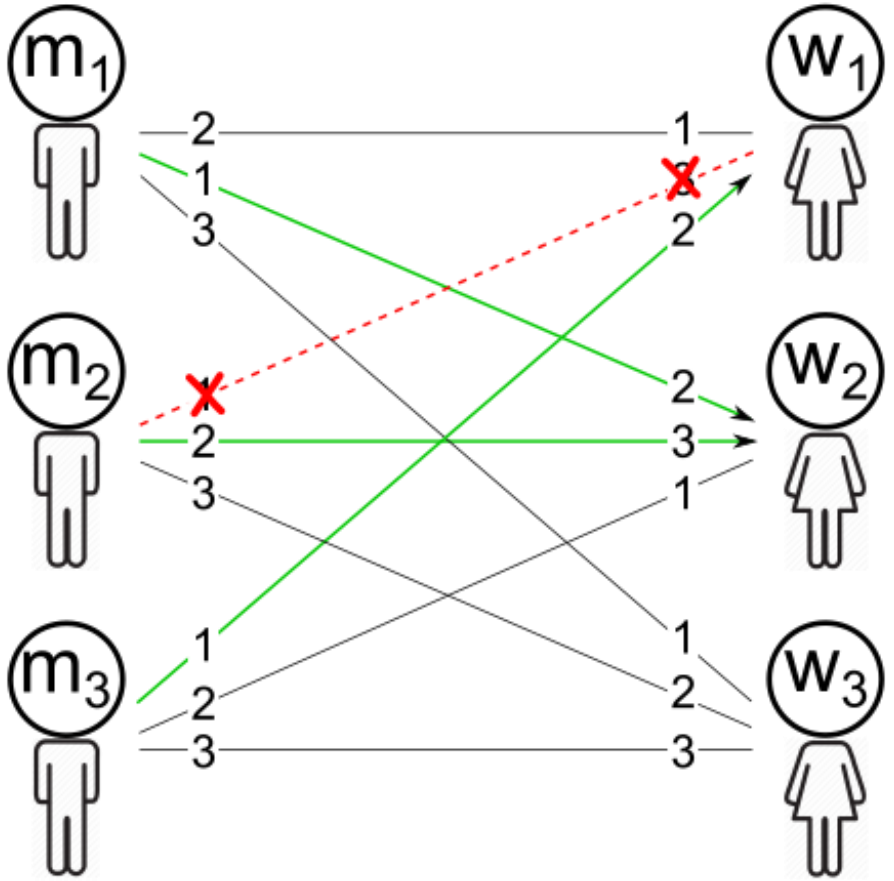

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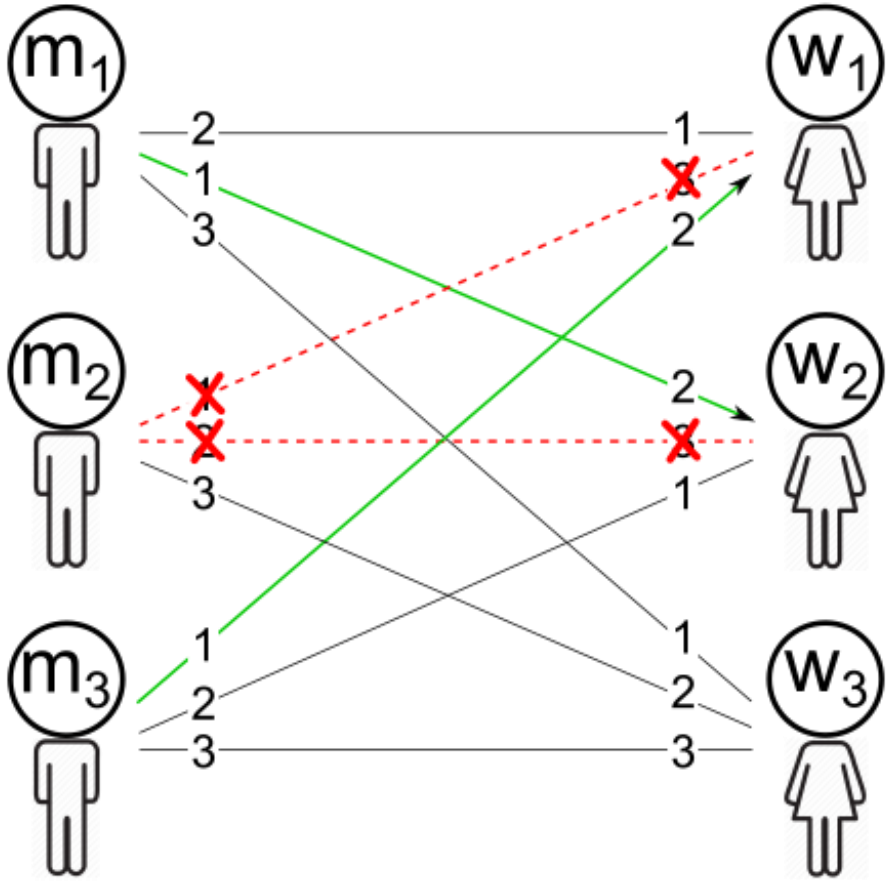

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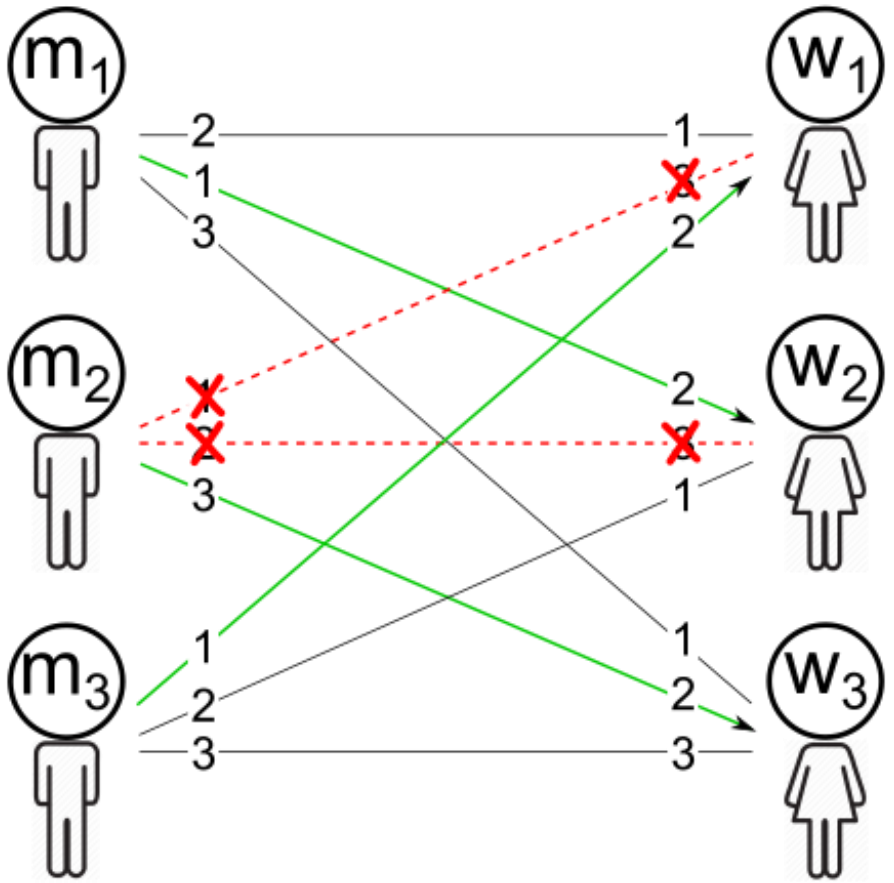

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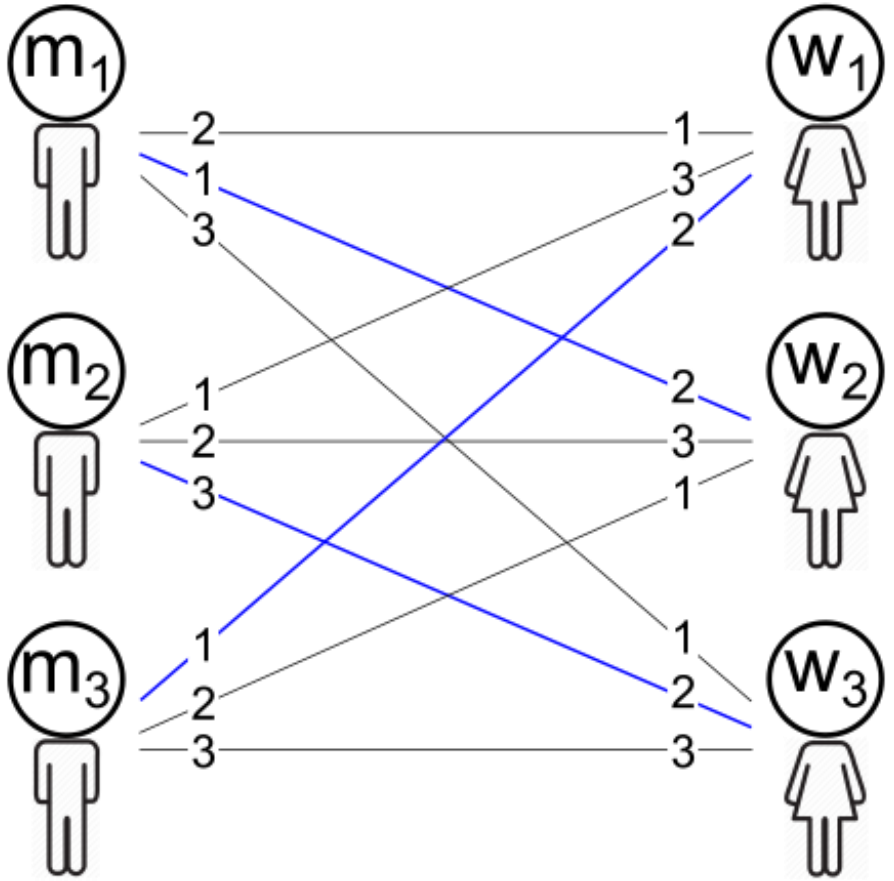

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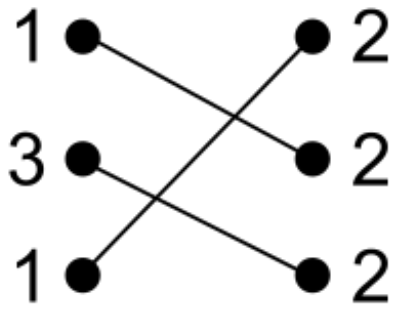
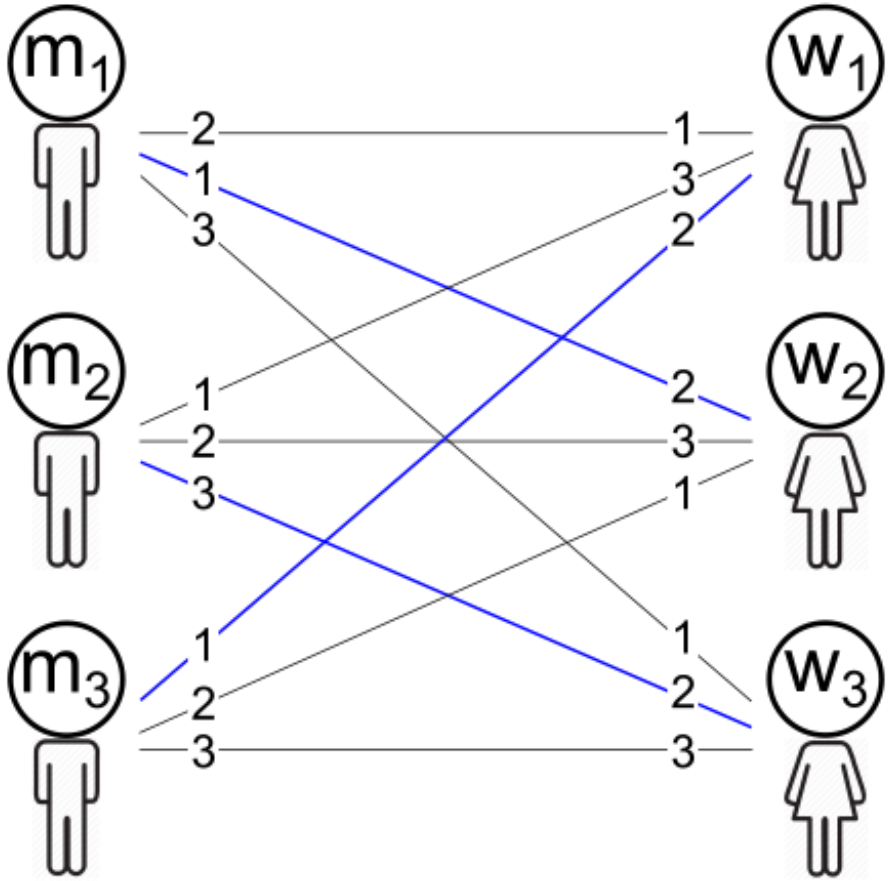
Outcome for true prefs





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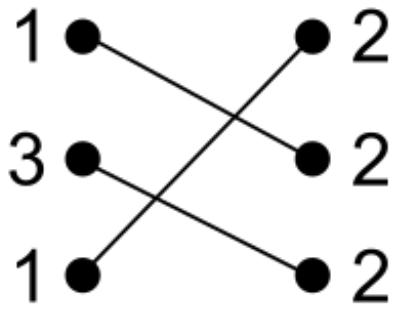

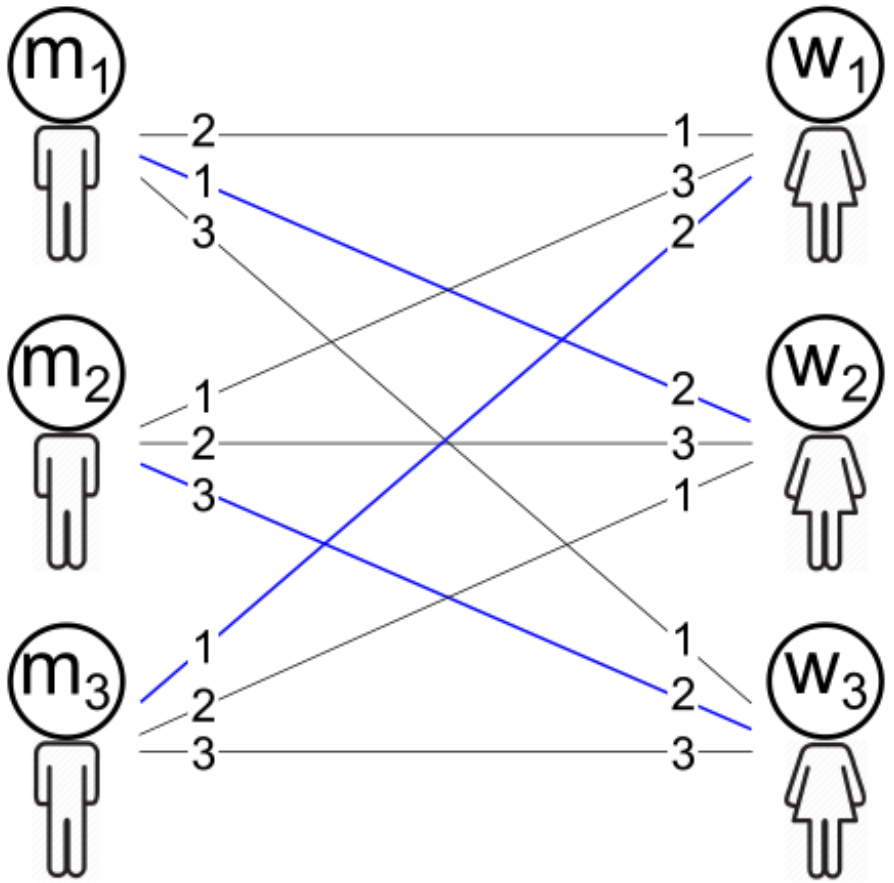

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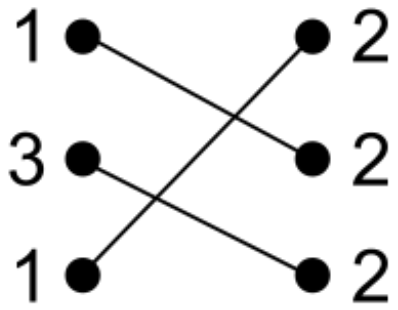

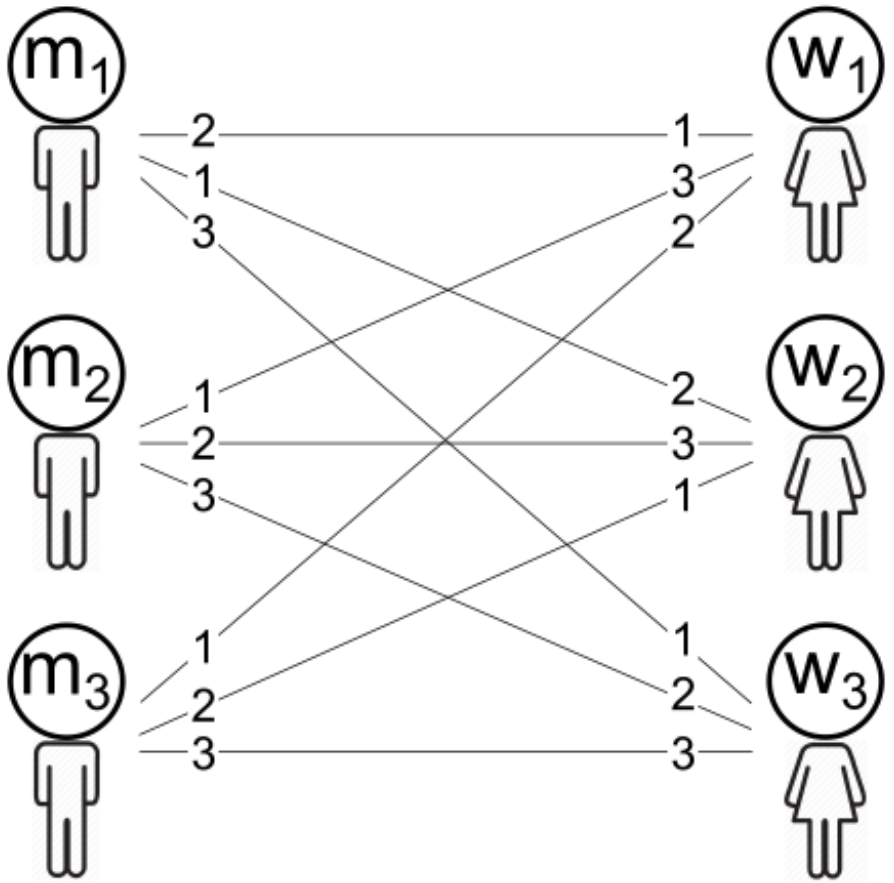

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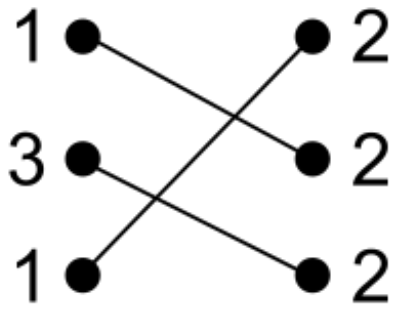

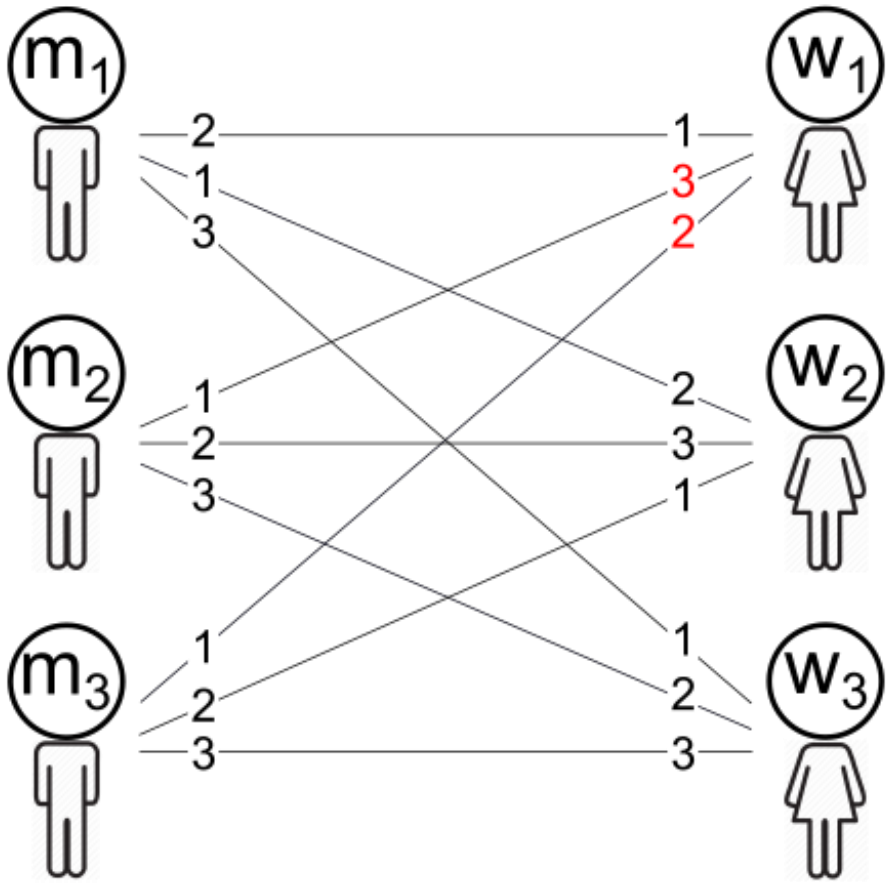

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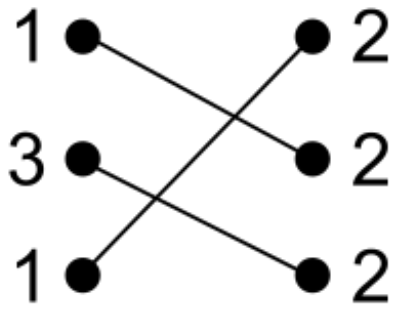

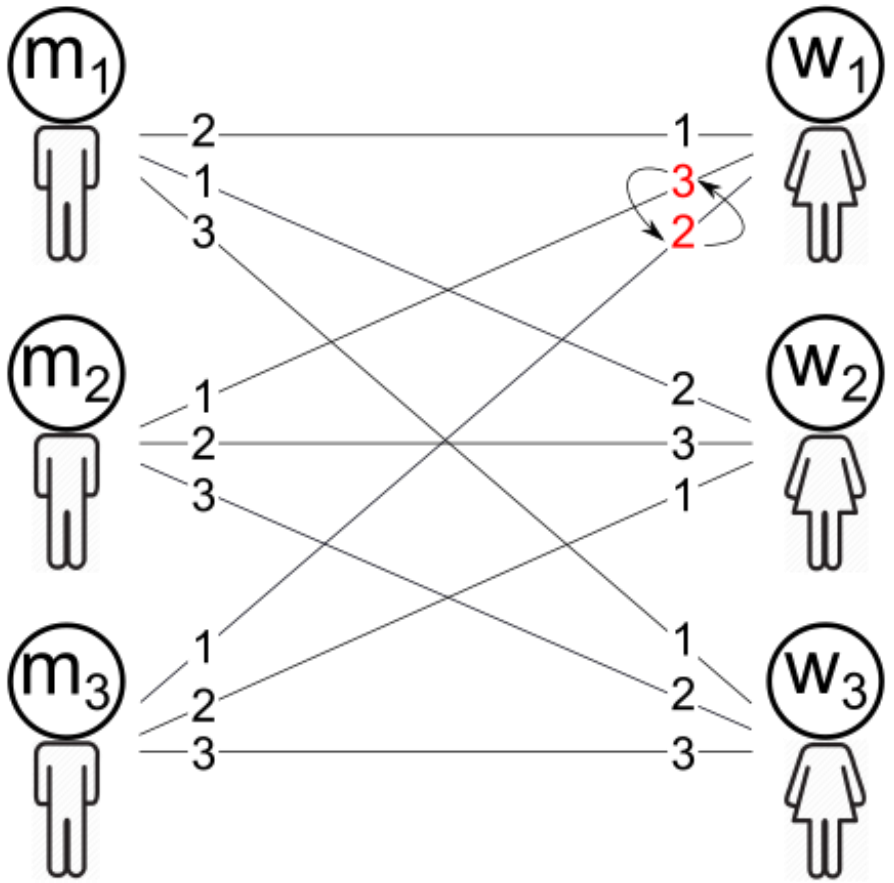

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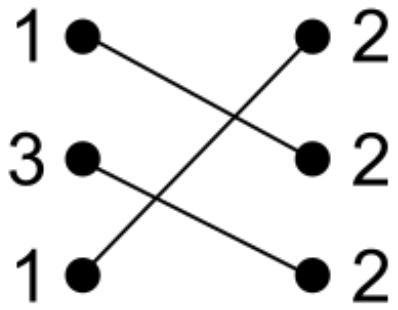

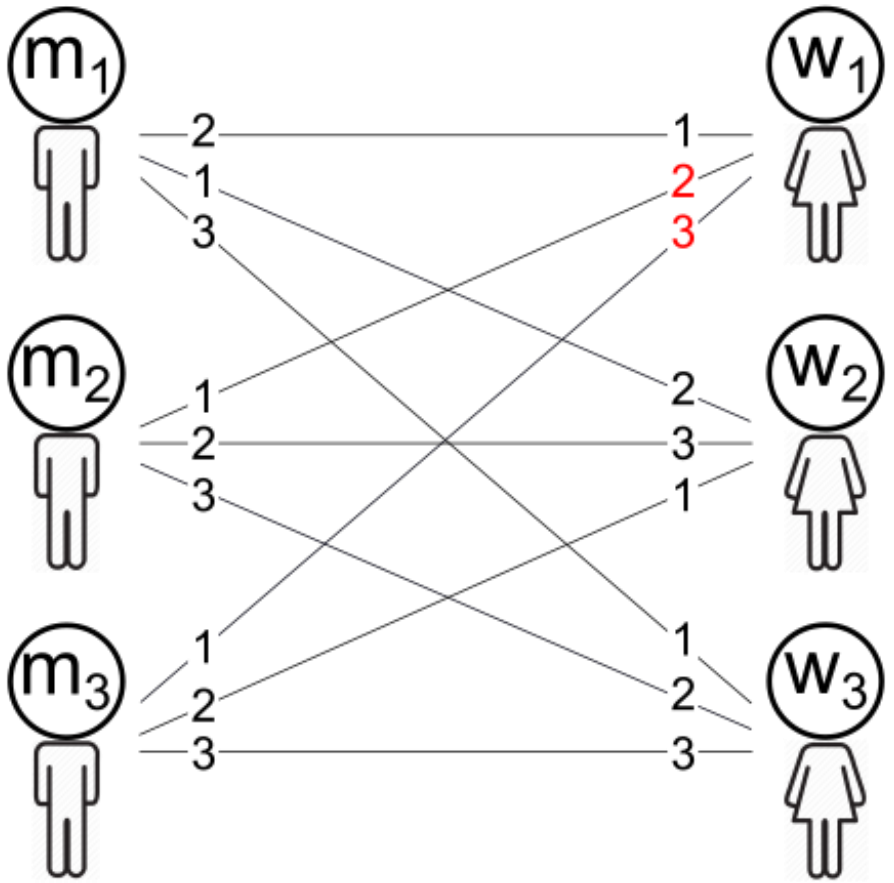

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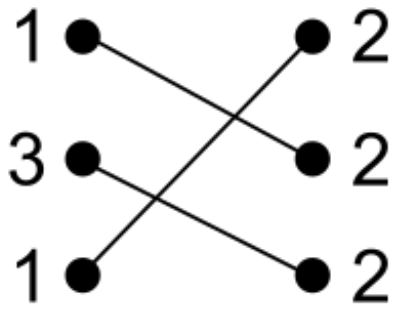

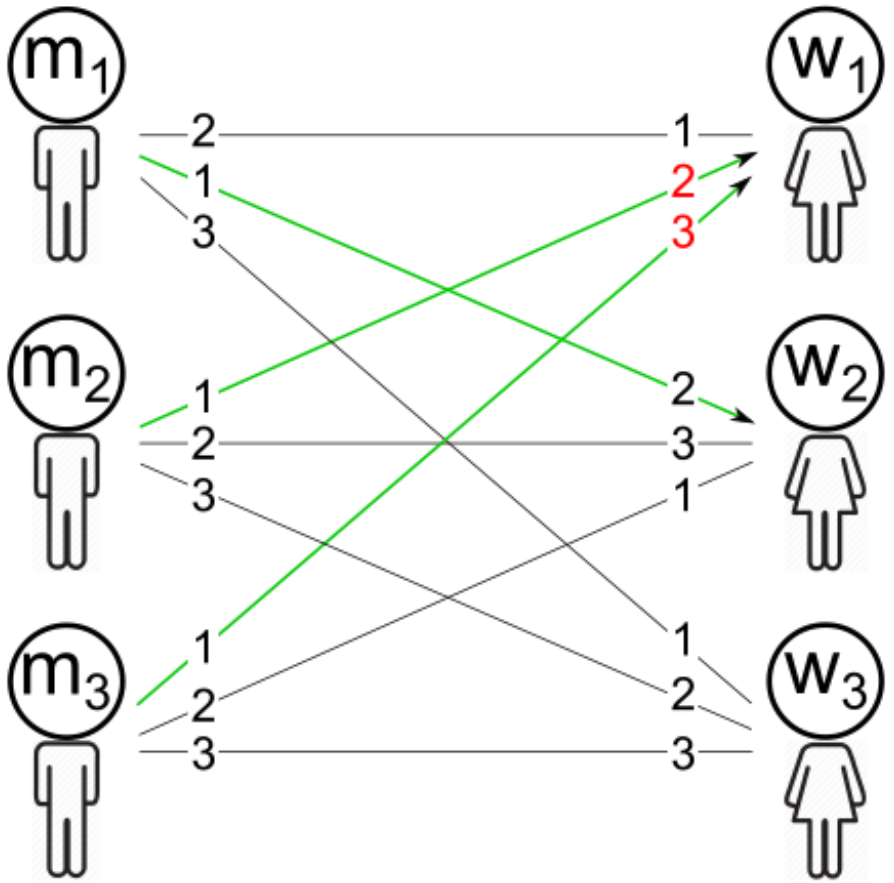

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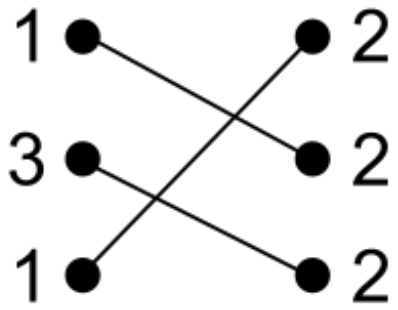

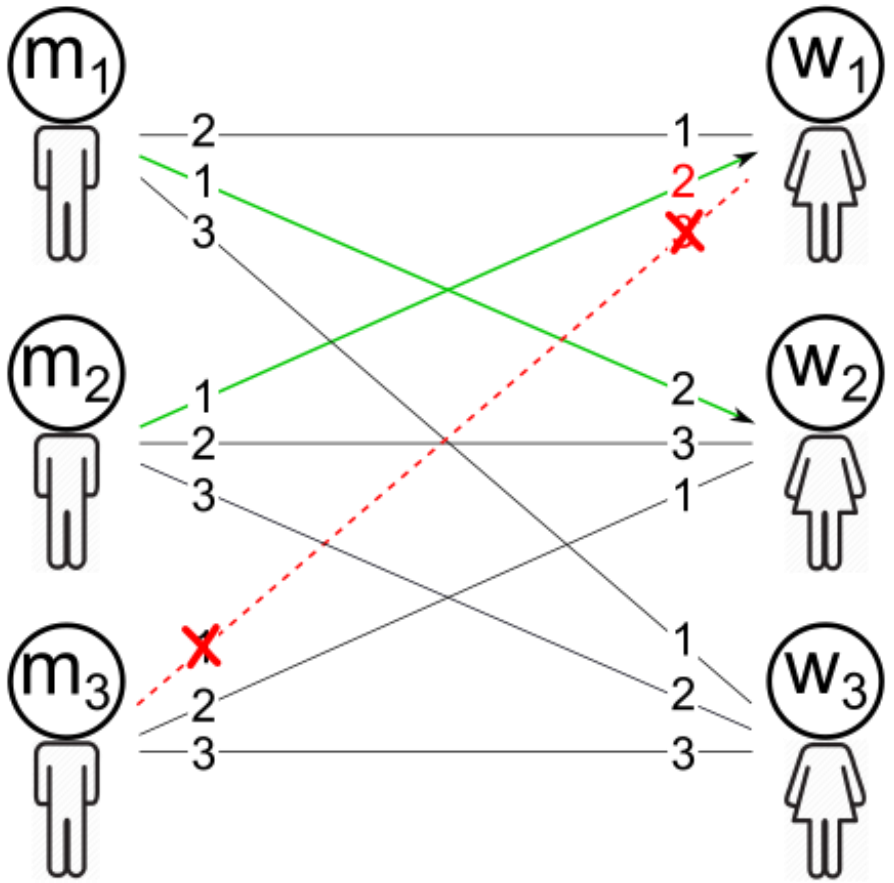

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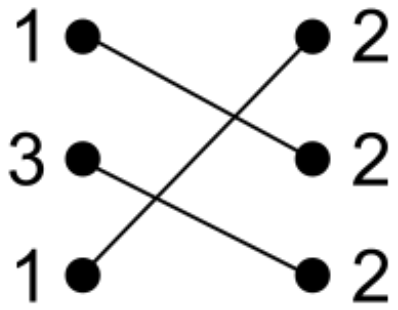

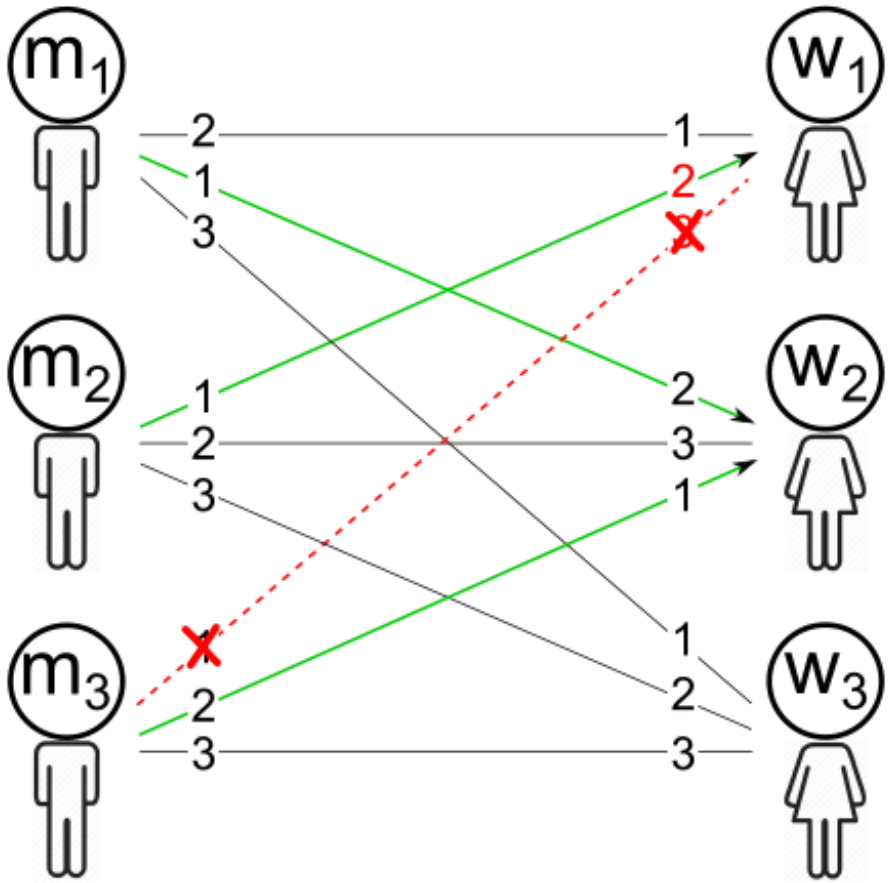

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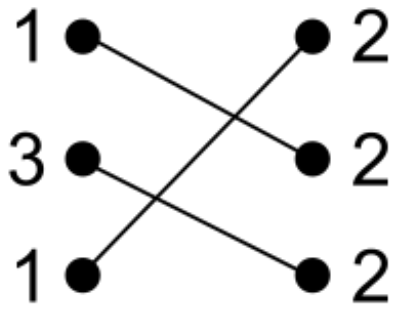

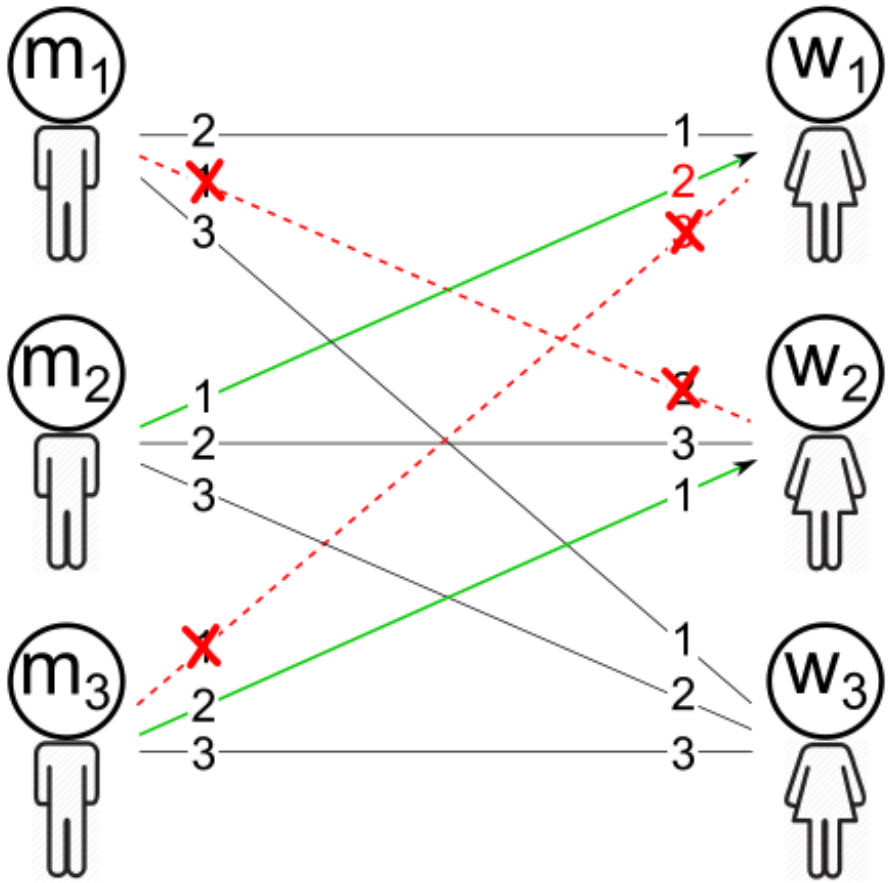

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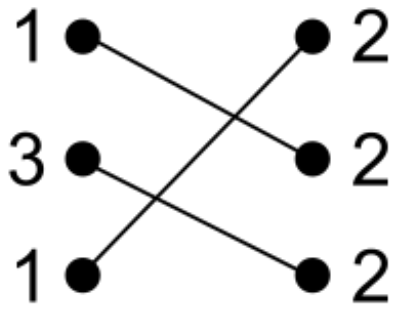

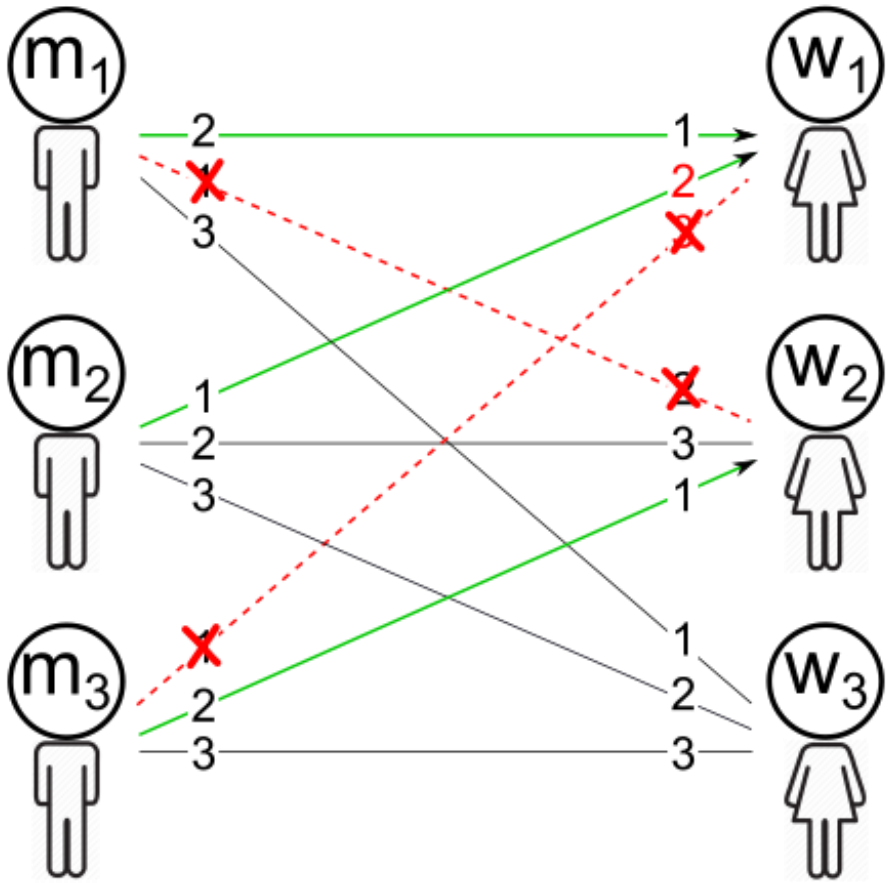

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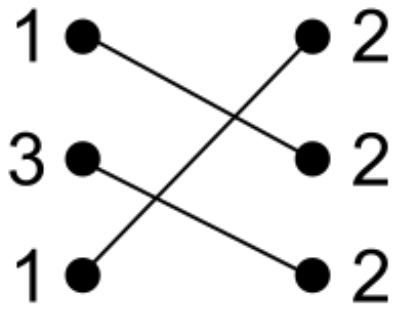

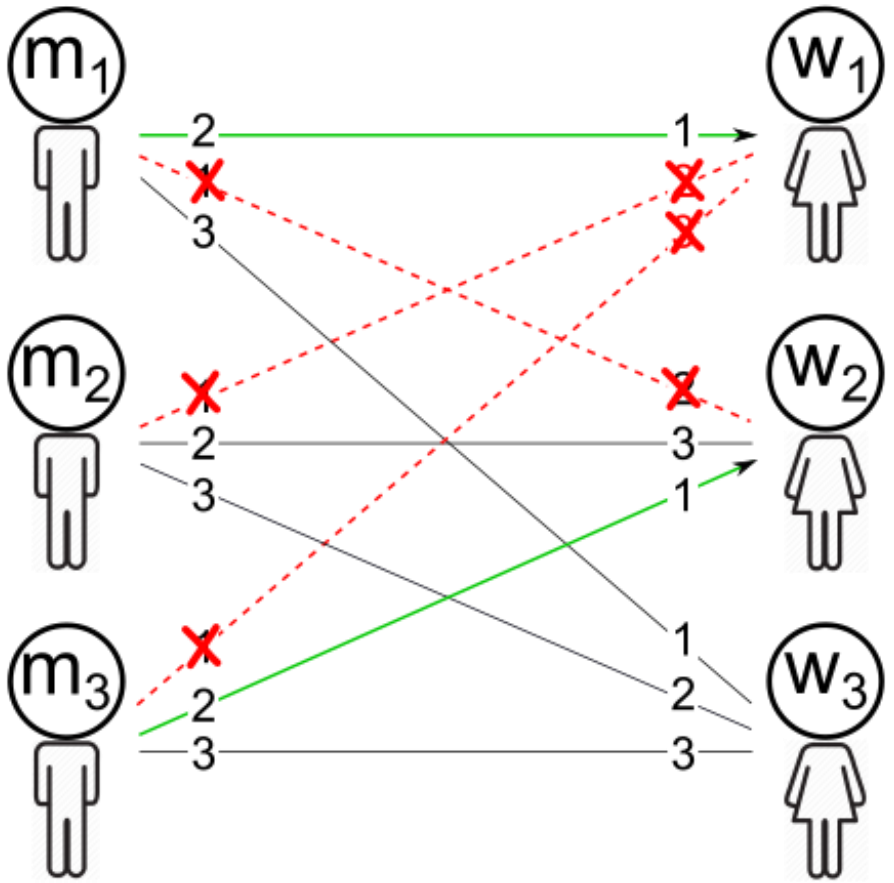

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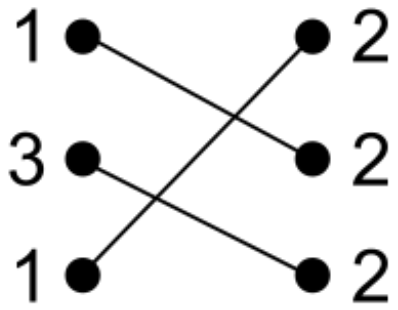

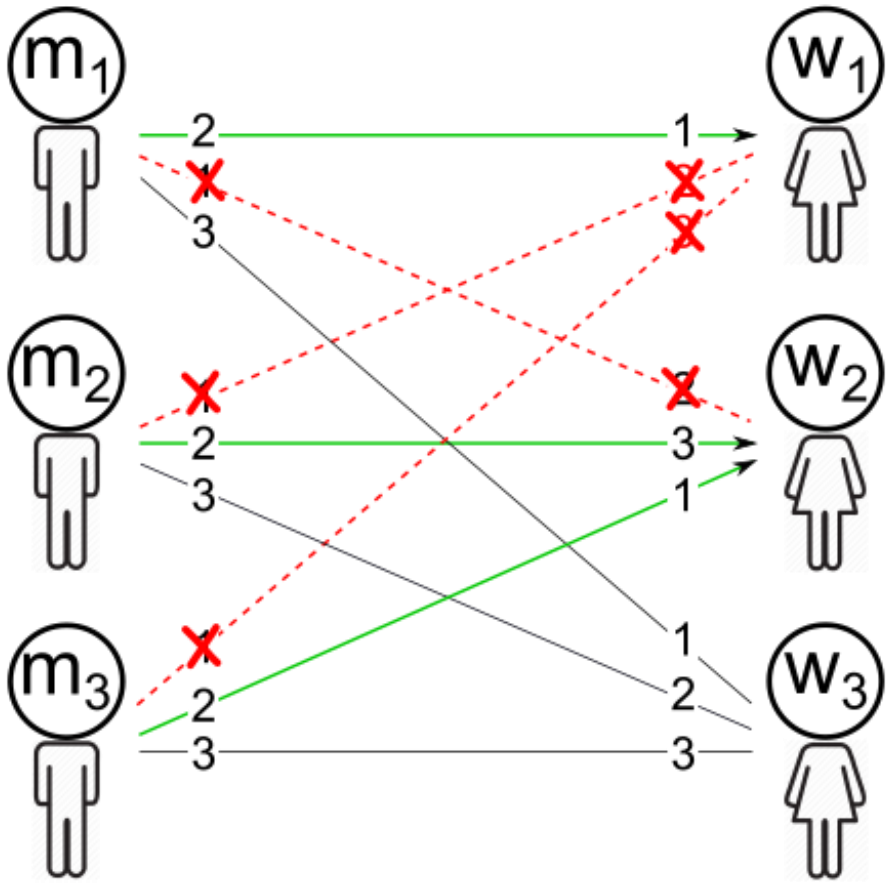

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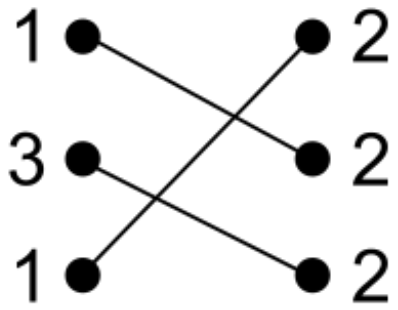

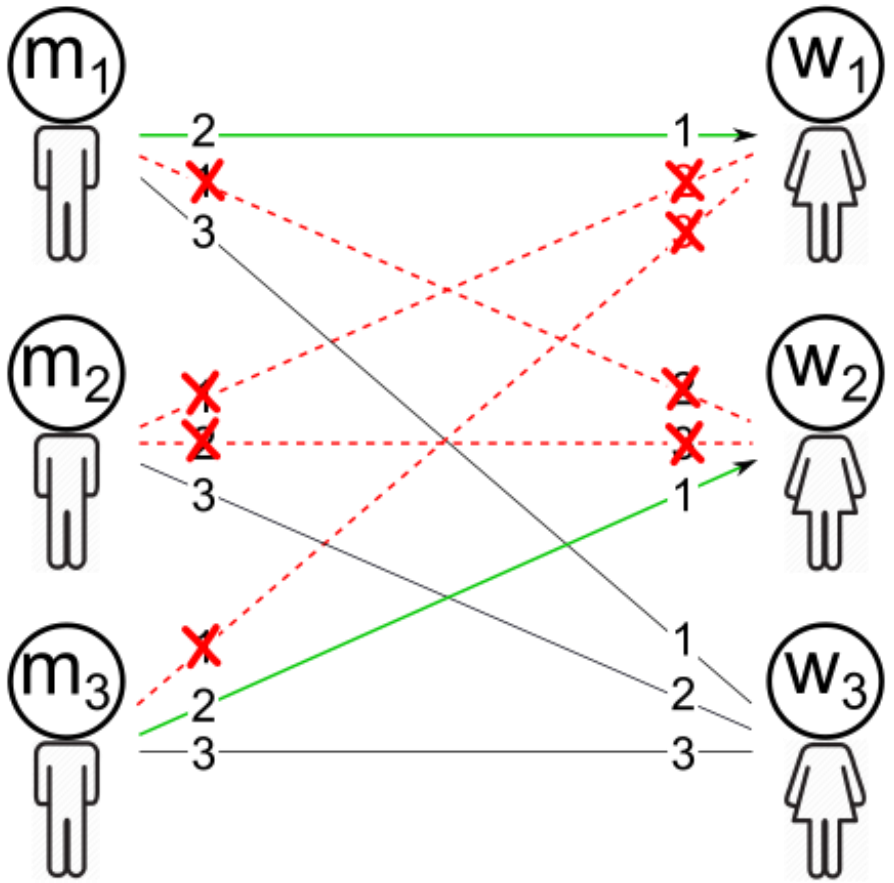

Can I get a better partner by misreporting my preferences?

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

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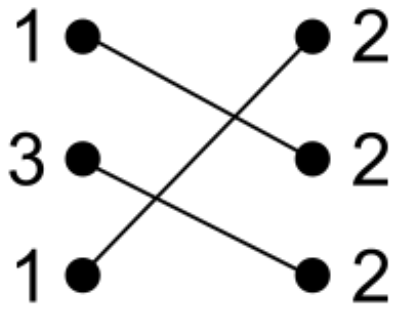

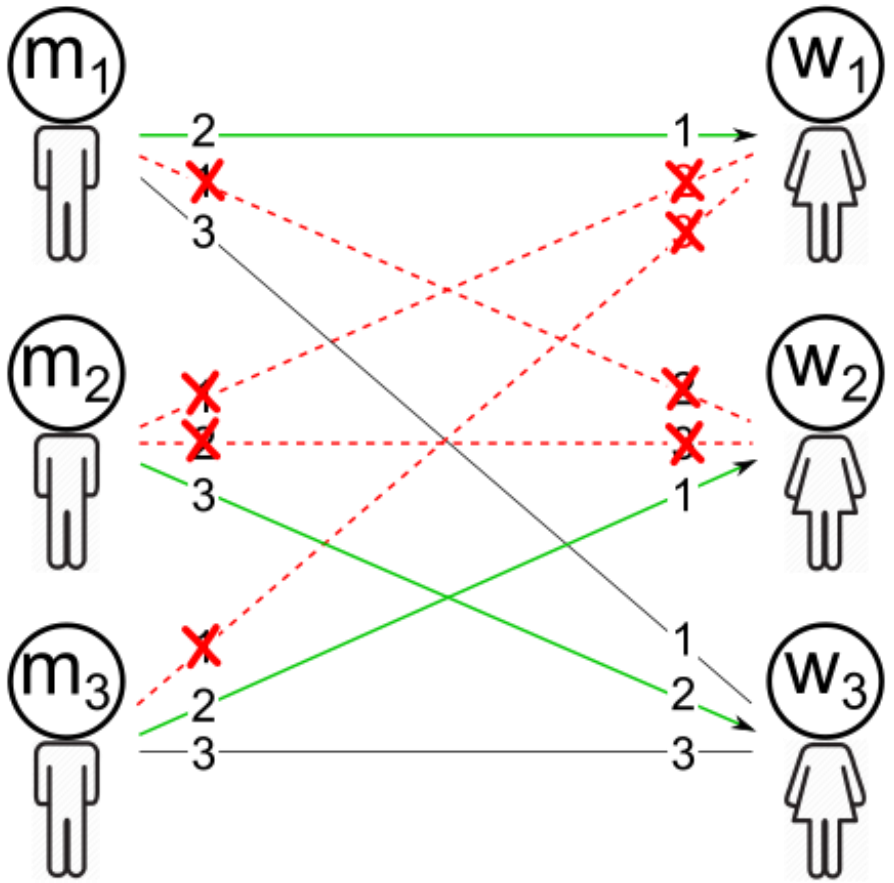

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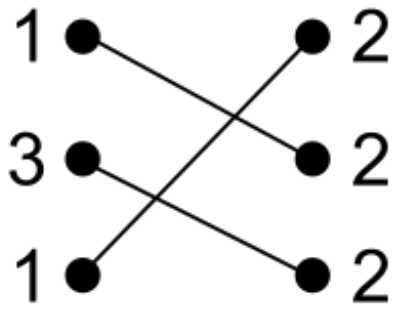

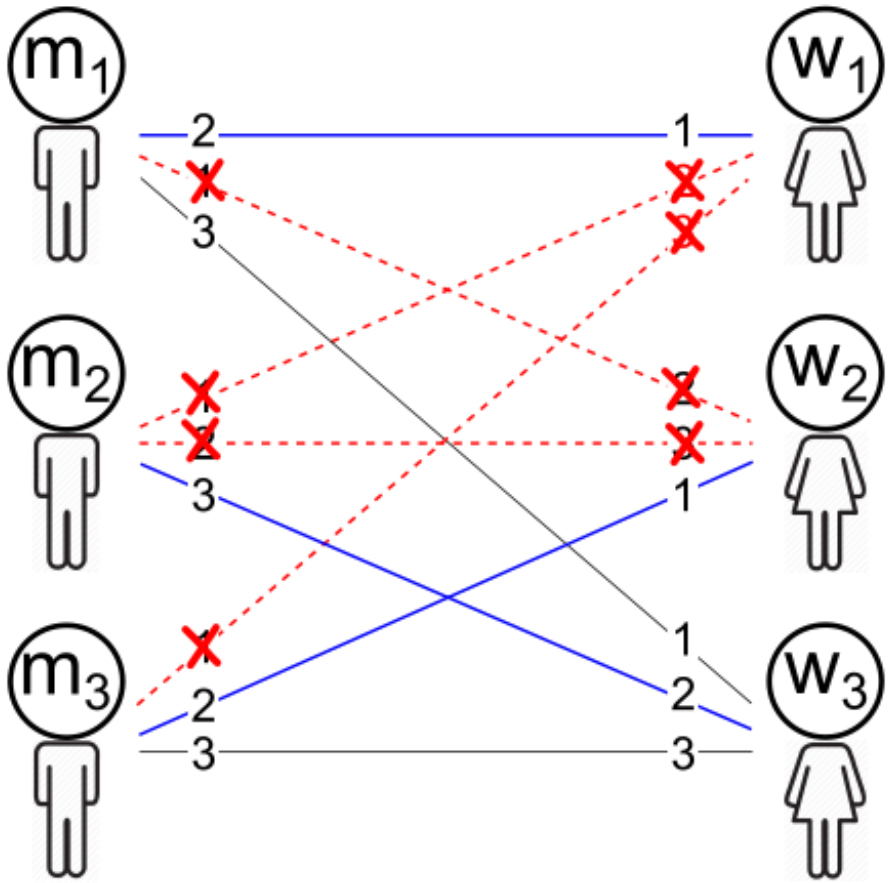

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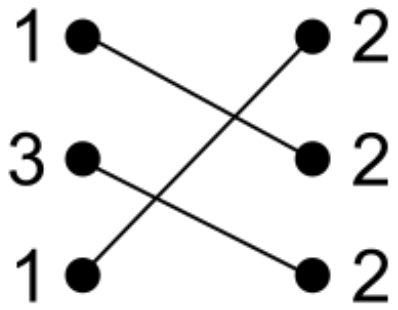

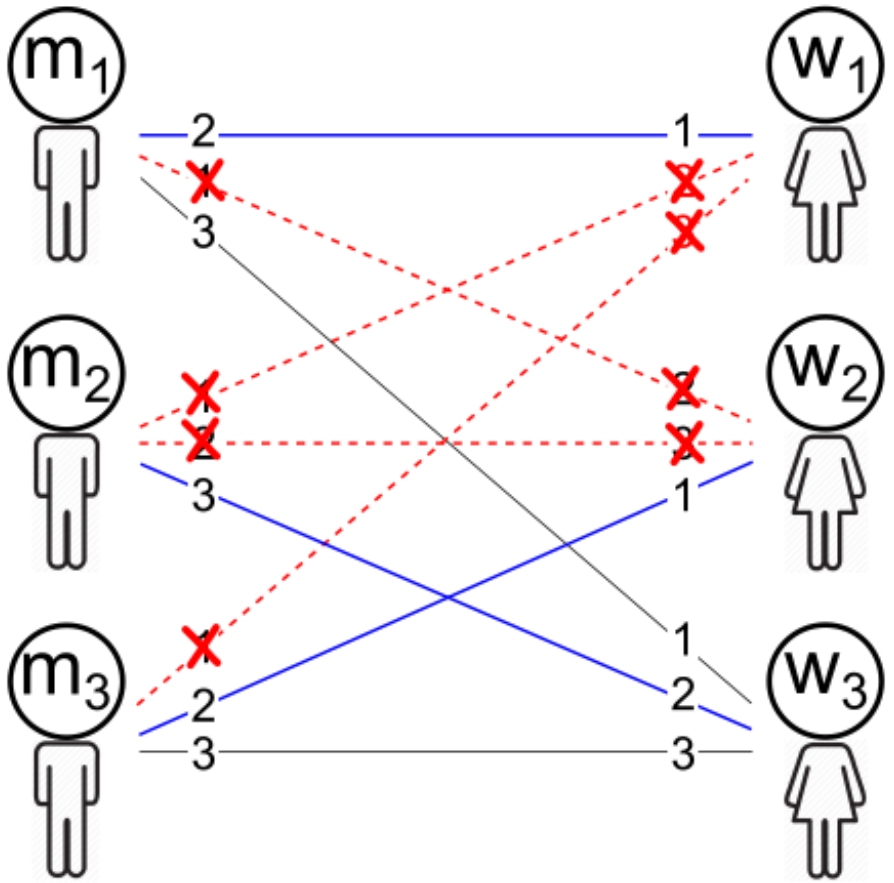

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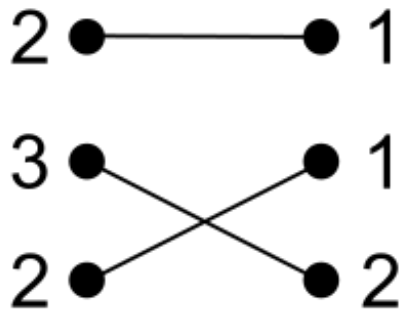

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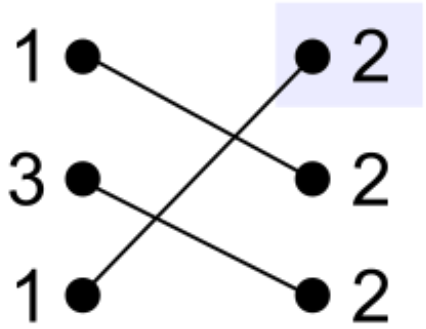

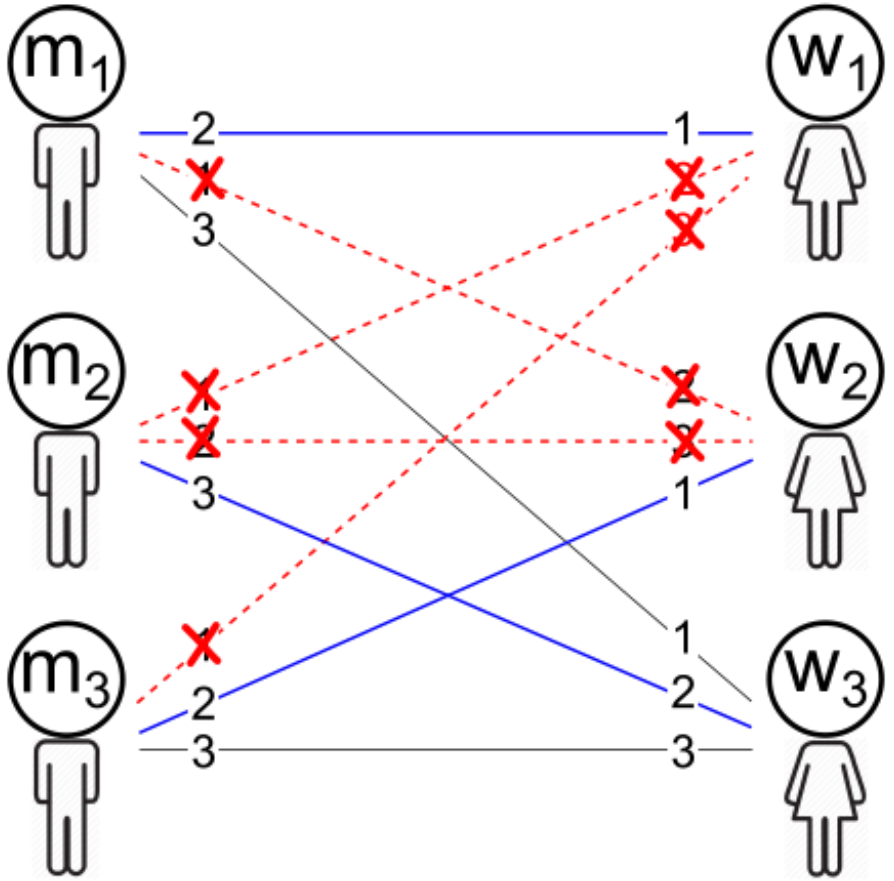

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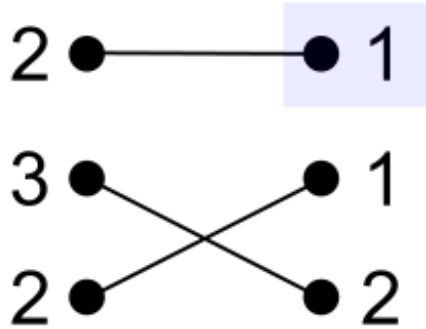

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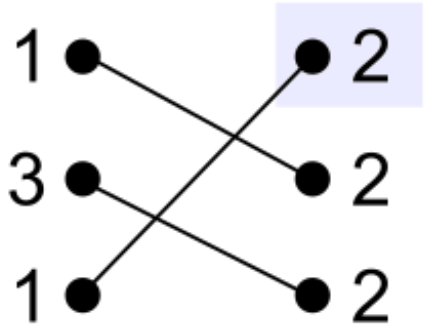
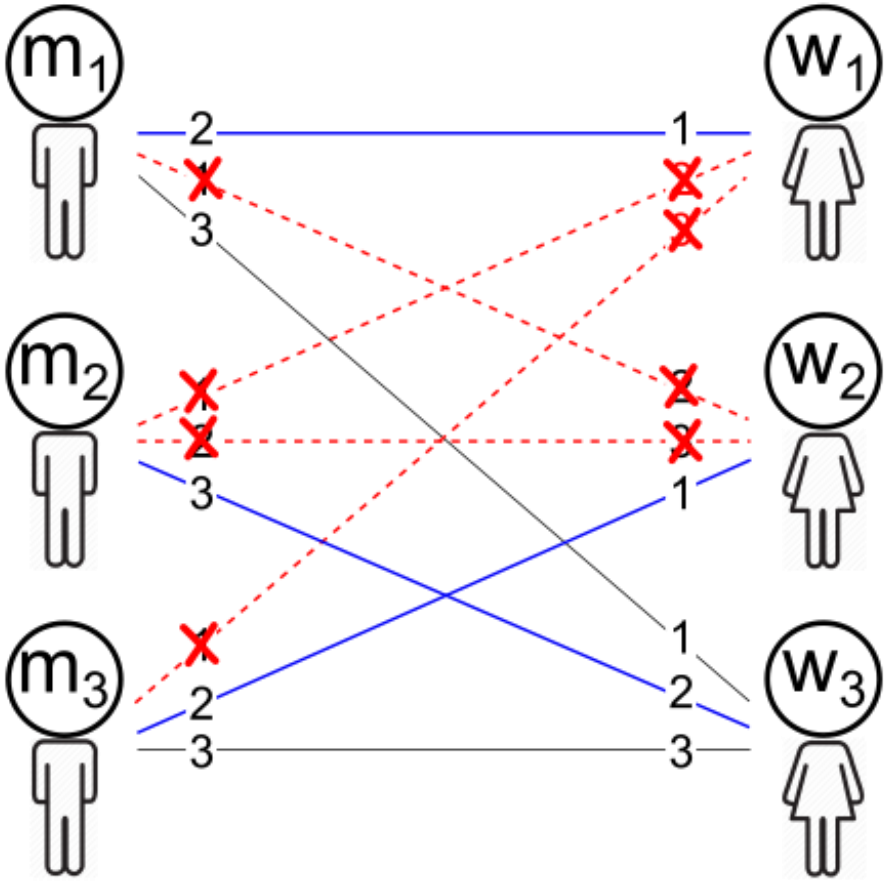
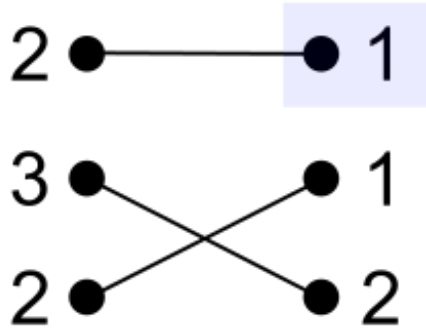




Yay!



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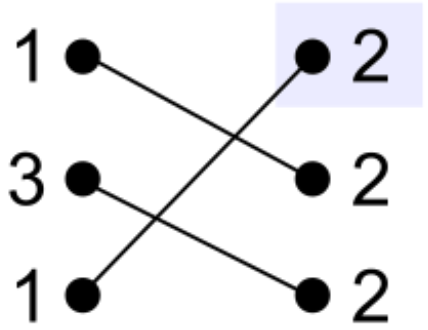

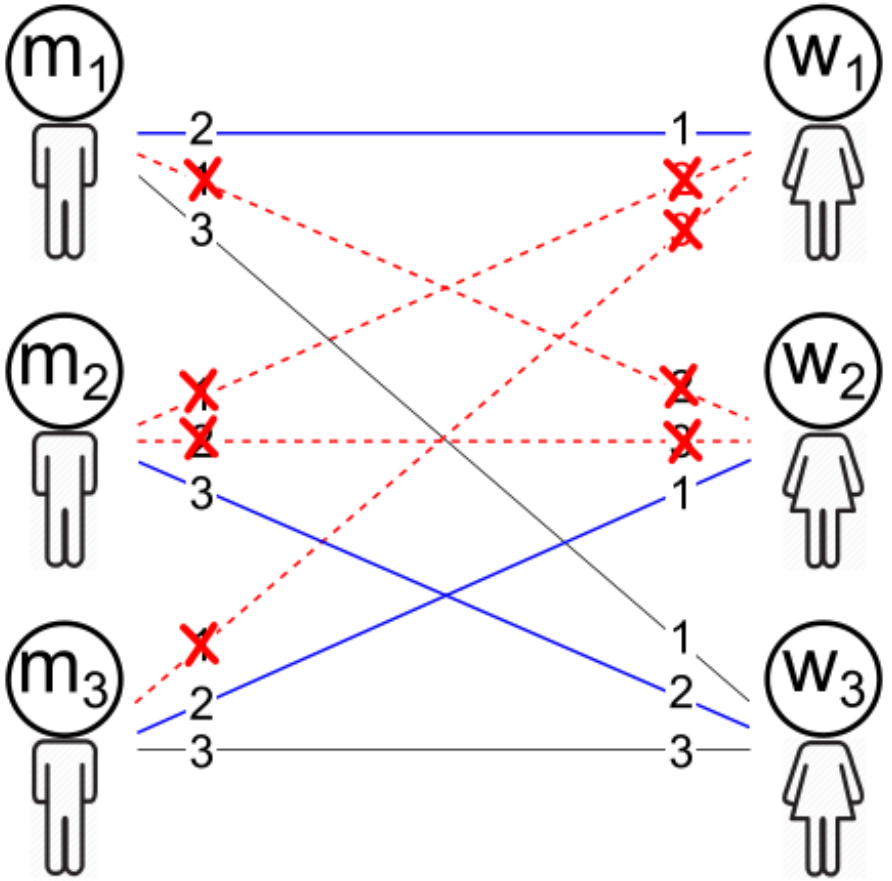

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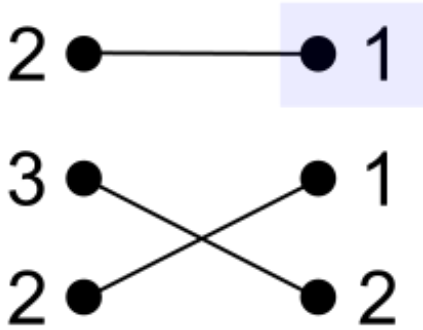

Any luck for the men?

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is strategyproof for the men.

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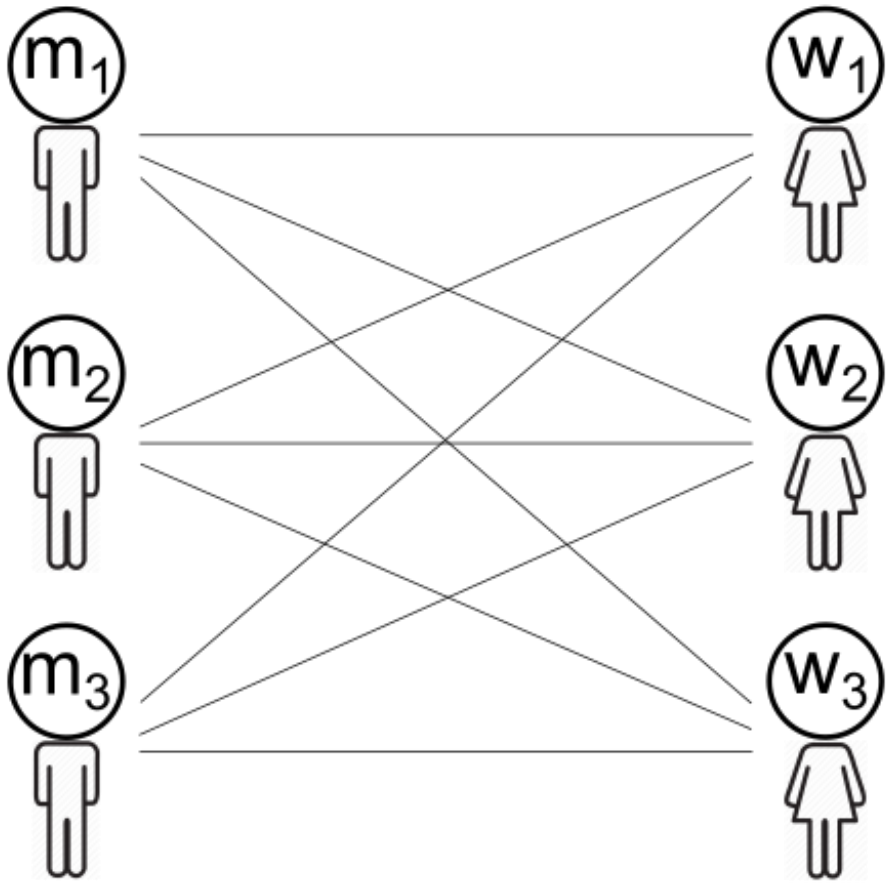

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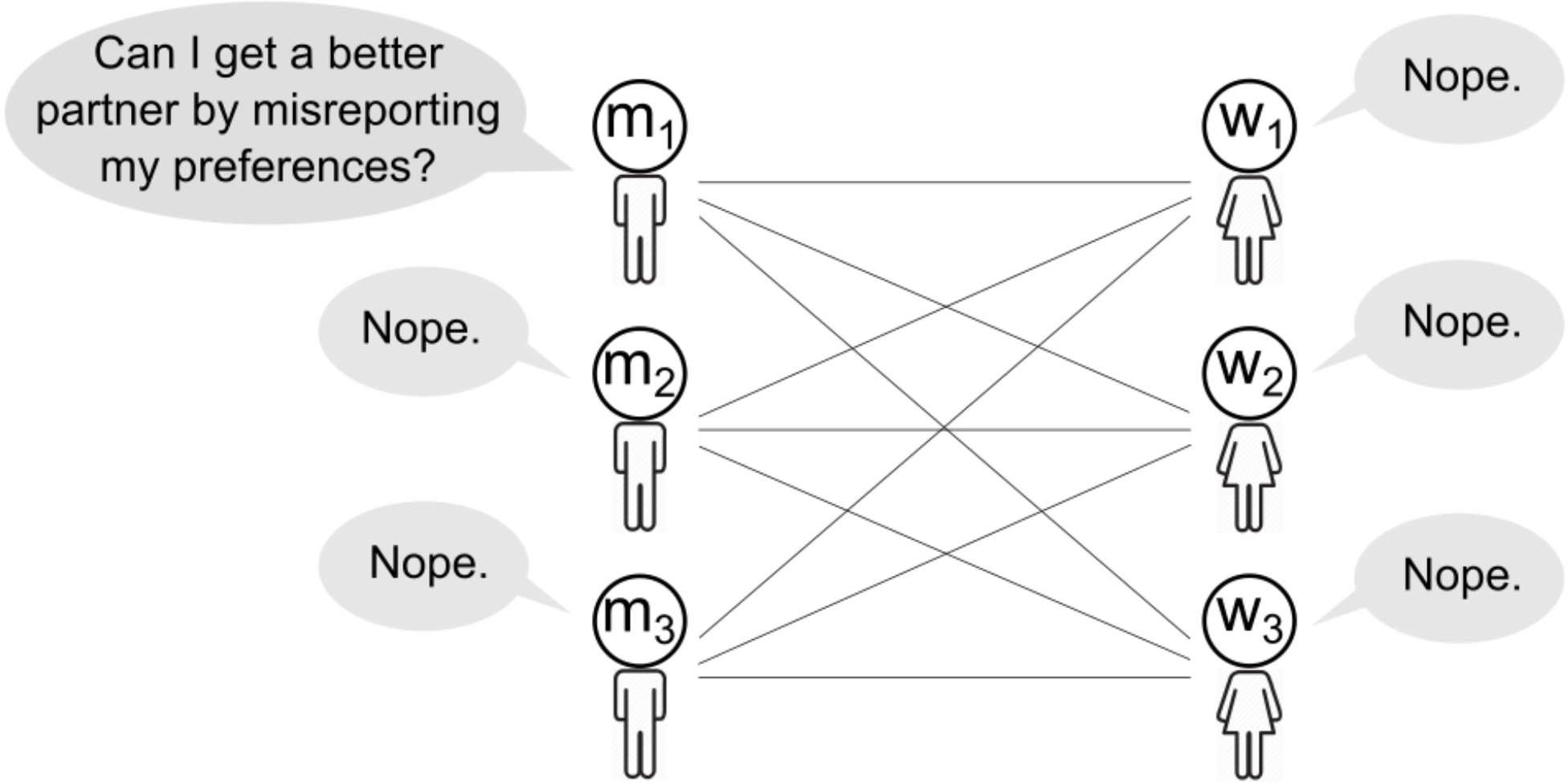

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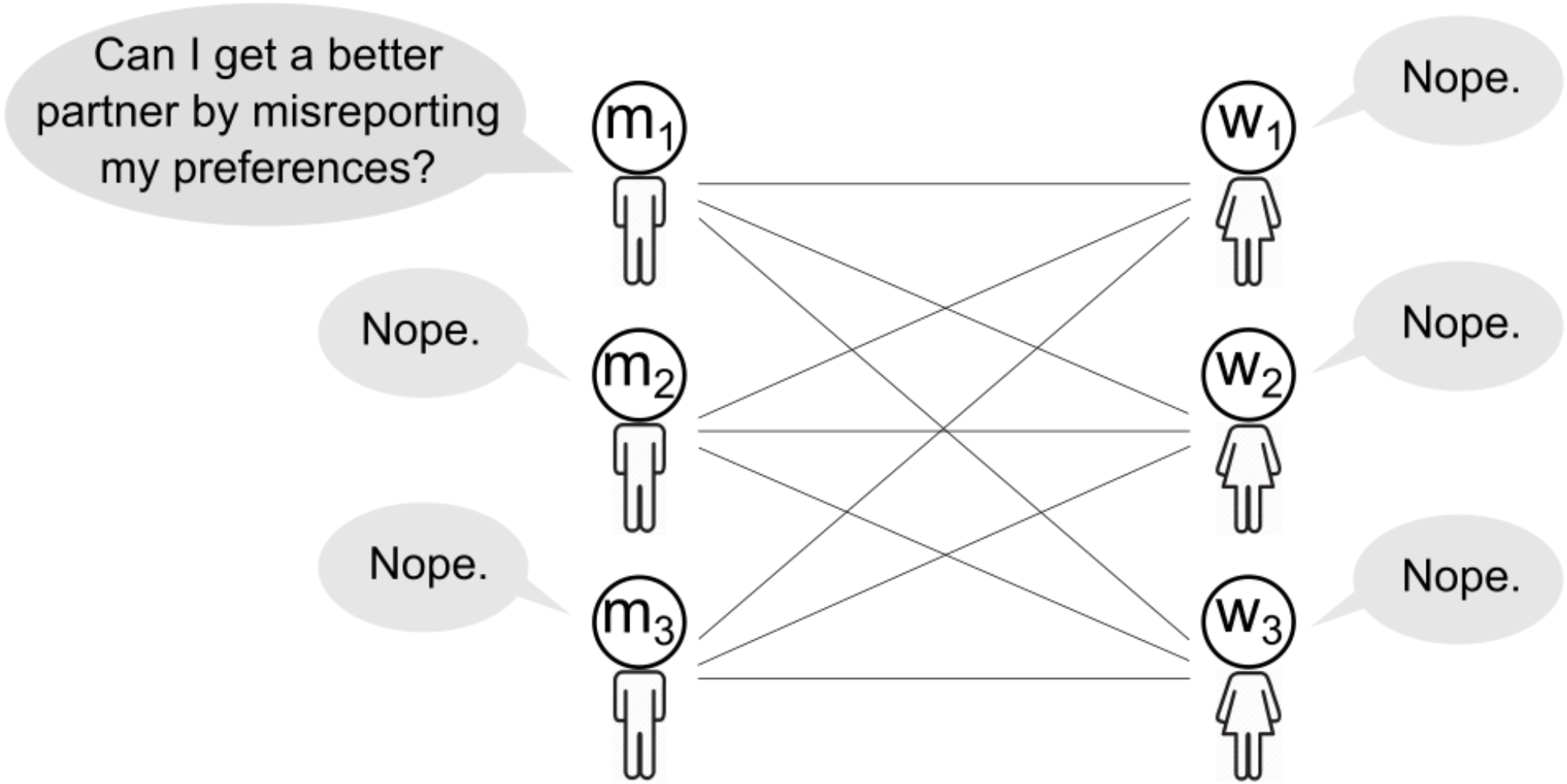

Proof ~~later in today's lecture~~ in the lecture slides.

So, men can't cheat in the men-proposing DA algorithm but women can.

Can we once again use computational hardness as a barrier to manipulation?

[Vaish and Garg, *IJCAI* 2017]

Any optimal manipulation for a woman can also be achieved by an "inconspicuous" misreport.

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Any optimal manipulation for a woman can also be achieved by an "inconspicuous" misreport.

True list of woman w: $m_1 > m_2 > m_3 > m_4 > \boxed{m_5} > m_6 > m_7 > m_8$

[Vaish and Garg, *IJCAI* 2017]

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An optimal manipulation: $\boxed{m_2} > m_4 > m_1 > m_8 > m_6 > m_3 > m_5 > m_7$

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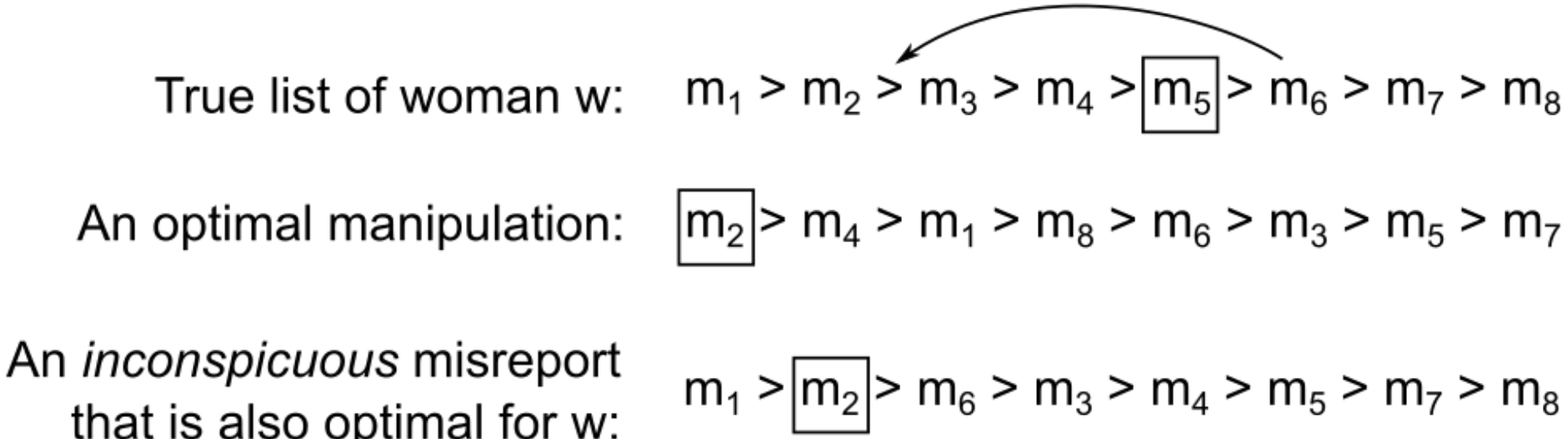
An *inconspicuous* misreport that is also optimal for w: $m_1 > \boxed{m_2} > m_6 > m_3 > m_4 > m_5 > m_7 > m_8$

[Vaish and Garg, *IJCAI* 2017]

Any optimal manipulation for a woman can also be achieved by an "inconspicuous" misreport.



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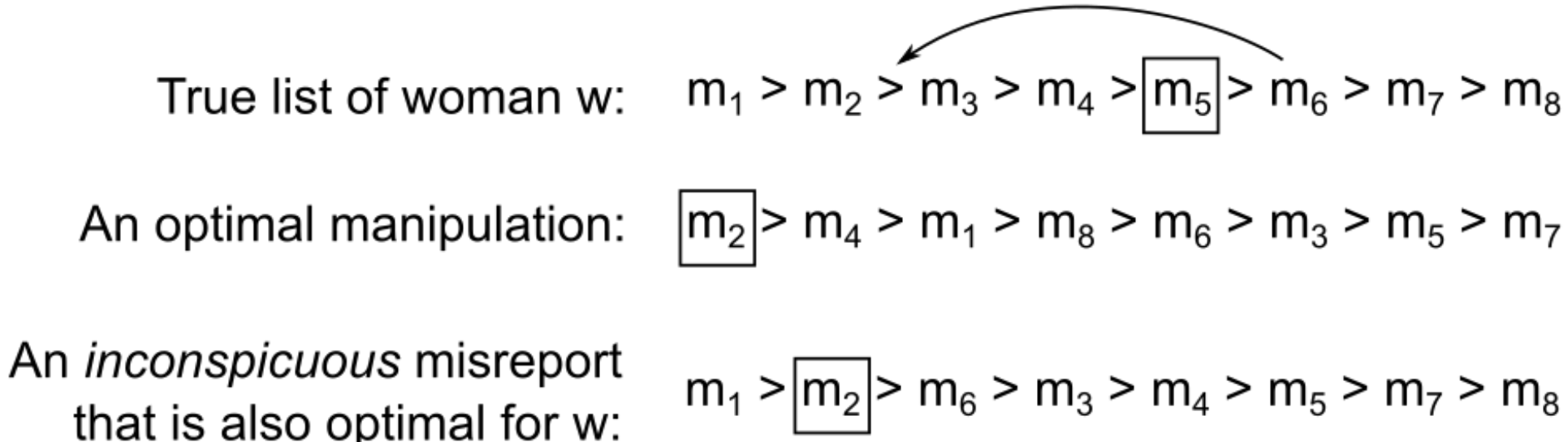
Any optimal manipulation for a woman can also be achieved by an "inconspicuous" misreport.



[Teo, Sethuraman and Tan, *Manag. Sci.* 2001; Vaish and Garg, *IJCAI* 2017]

An optimal manipulation for a woman can be computed in polynomial time.

But there is a saving grace...

[Vaish and Garg, *IJCAI* 2017]

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

[Vaish and Garg, *IJCAI* 2017]

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

We will use the following observation:

Suppose a woman promotes a man m in her list
and no other changes are made.
If m proposed to her during DA on the old profile,
then he proposes to her during DA on the new profile.

[Vaish and Garg, *IJCAI* 2017]

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Idea: Any deviation between old and new runs of the DA must involve tentative acceptance/rejection of m, but that can happen only after m proposes.

[Vaish and Garg, *IJCAI* 2017]

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P = profile with true preferences

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It must be that w' = w.

If w' ≠ w, then m and w' are both truthful and will block X' w.r.t. P'---contradicting the stability of DA.

[Vaish and Garg, *IJCAI* 2017]

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So, (m,w) blocks X' w.r.t. P.

P_m	P_w
•	•
•	•
w	m
•	•
•	•
X'(m)	X'(w)
•	•
•	•

[Vaish and Garg, *IJCAI* 2017]

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P_m	P_w	P'_m	P'_w
•	•	•	•
•	•	•	•
w	m	w	X'(w)
•	•	•	•
•	•	•	•
•	•	•	•
X'(m)	X'(w)	X'(m)	m
•	•	•	•
•	•	•	•

[Vaish and Garg, *IJCAI* 2017]

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P_m	P_w	P'_m	P'_w
•	•	•	•
•	•	•	•
w	m	w	X'(w)
•	•	•	•
•	•	•	•
•	•	•	•
X'(m)	X'(w)	X'(m)	m
•	•	•	•
•	•	•	•

m must propose to w during DA(P')

[Vaish and Garg, *IJCAI* 2017]

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So, (m,w) blocks X' w.r.t. P.

P_m	P_w	P'_m	P'_w	P''_m	P''_w
•	•	•	•	•	m
•	•	•	•	•	•
w	m	w	X'(w)	w	•
•	•	•	•	•	•
•	•	•	•	•	X'(w)
•	•	•	•	•	•
X'(m)	X'(w)	X'(m)	m	X'(m)	•
•	•	•	•	•	•
•	•	•	•	•	•

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Recall:

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So, (m,w) blocks X' w.r.t. P.

P_m	P_w	P'_m	P'_w	P''_m	P''_w
•	•	•	•	•	m
•	•	•	•	•	•
w	m	w	X'(w)	w	•
•	•	•	•	•	•
•	•	•	•	•	X'(w)
•	•	•	•	•	•
X'(m)	X'(w)	X'(m)	m	X'(m)	•
•	•	•	•	•	•
•	•	•	•	•	•

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P_m	P_w	P'_m	P'_w	P''_m	P''_w
•	•	•	•	•	m
•	•	•	•	•	•
w	m	w	X'(w)	w	•
•	•	•	•	•	•
•	•	•	•	•	X'(w)
•	•	•	•	•	•
X'(m)	X'(w)	X'(m)	m	X'(m)	•
•	•	•	•	•	•
•	•	•	•	•	•

m must propose to w during DA(P')

⇒m must propose to w during DA(P'')

[Vaish and Garg, *IJCAI* 2017]

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So, (m,w) blocks X' w.r.t. P.

P_m	P_w	P'_m	P'_w	P''_m	P''_w
•	•	•	•	•	m
•	•	•	•	•	•
w	m	w	X'(w)	w	•
•	•	•	•	•	•
•	•	•	•	•	X'(w)
•	•	•	•	•	•
X'(m)	X'(w)	X'(m)	m	X'(m)	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•

m must propose to w during DA(P')

⇒m must propose to w during DA(P'')

⇒X''(w) = m, where X''=DA(P'')

[Vaish and Garg, *IJCAI* 2017]

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P' = profile with manipulated preferences

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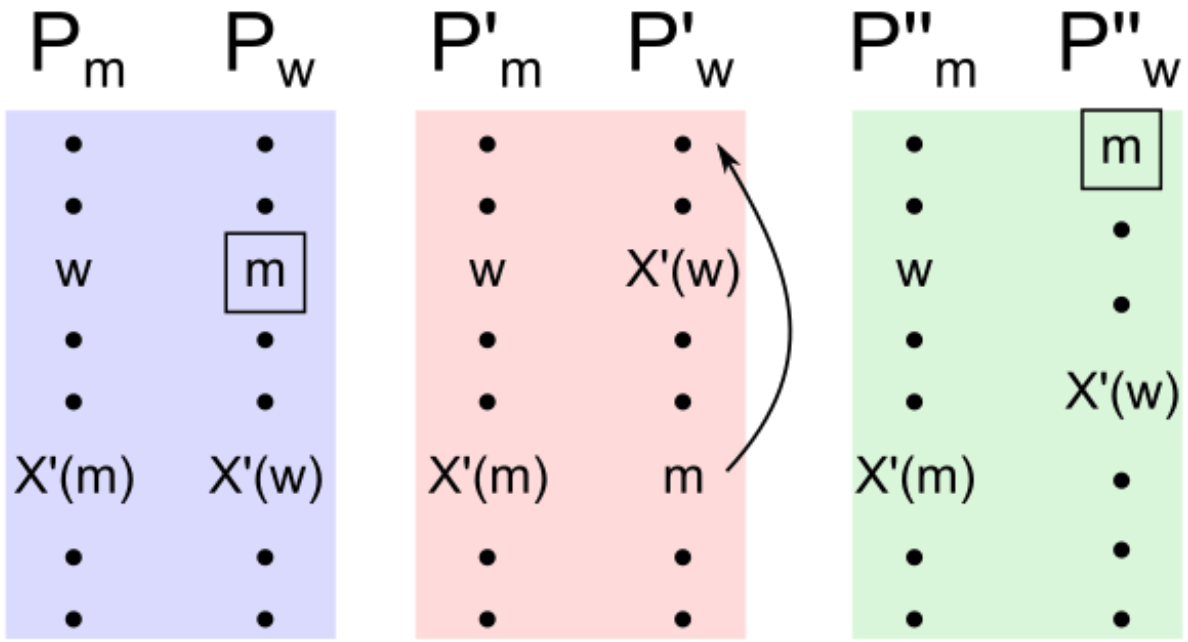
So, (m,w) blocks X' w.r.t. P.



m must propose to w during DA(P')

⇒m must propose to w during DA(P")

⇒X"(w) = m, where X"=DA(P")

[Vaish and Garg, *IJCAI* 2017]

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

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$P = (P_{-w}, P_w)$

X = DA(P)

P' = profile with manipulated preferences

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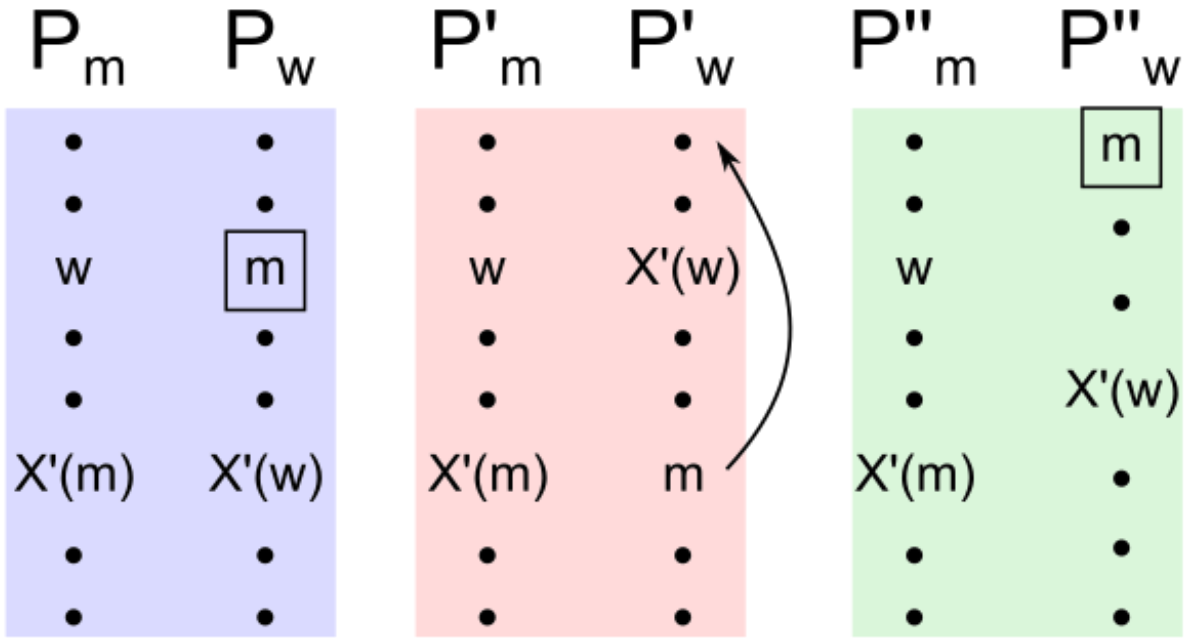
So, (m,w) blocks X' w.r.t. P.



m must propose to w during DA(P')

⇒m must propose to w during DA(P")

⇒X"(w) = m, where X"=DA(P")

But then, P''_w gives w a better partner than under her optimal strategy!

[Vaish and Garg, *IJCAI* 2017]

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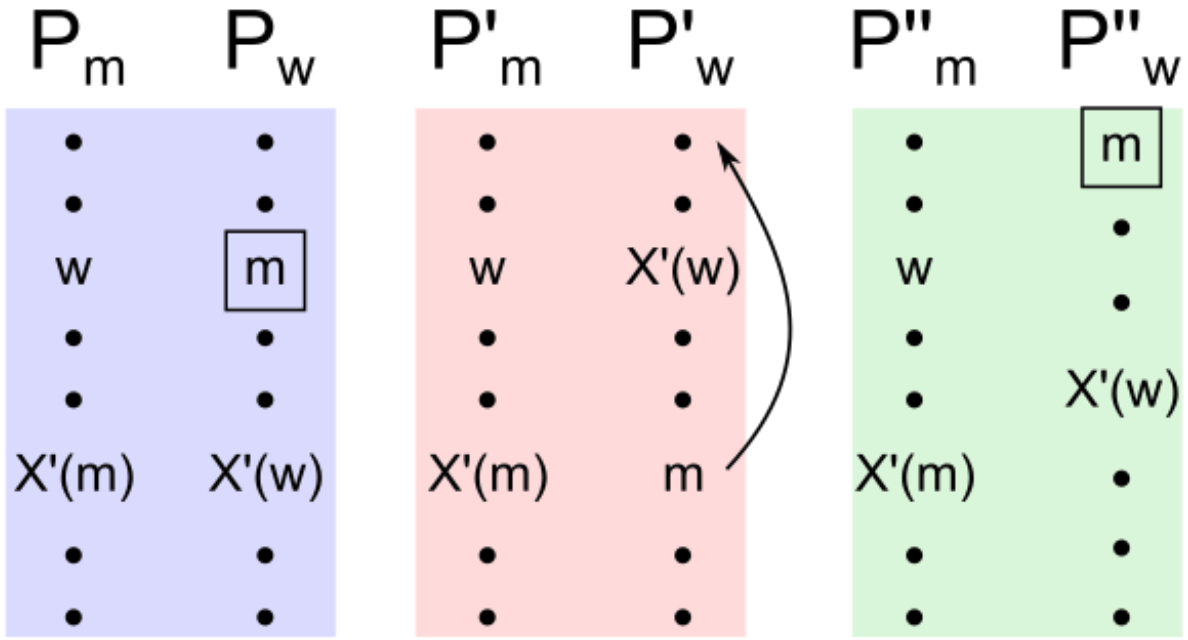
X' = DA(P')

So, (m,w) blocks X' w.r.t. P.



m must propose to w during DA(P')

⇒m must propose to w during DA(P")

⇒X"(w) = m, where X"=DA(P")

But then, P''_w gives w a better partner than under her optimal strategy!

■

[Vaish and Garg, *IJCAI* 2017]

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

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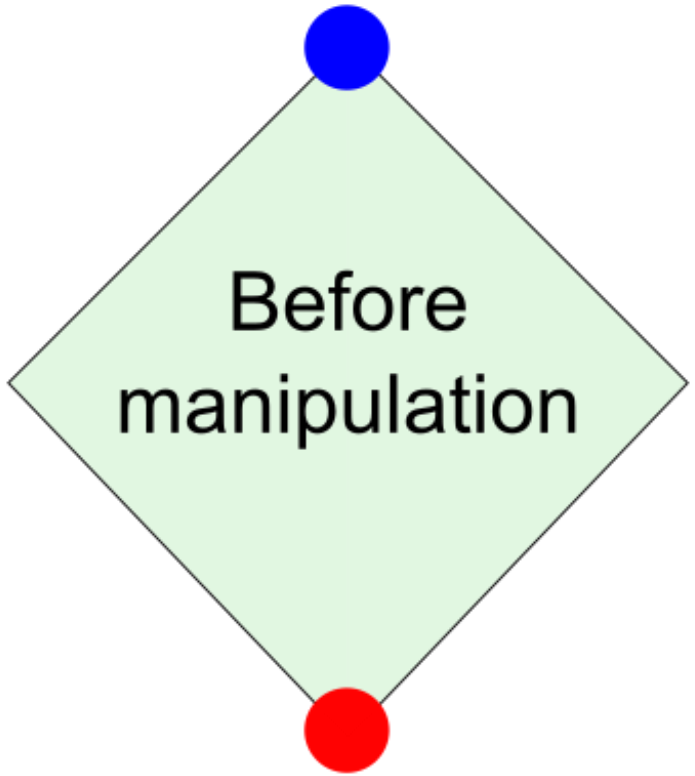

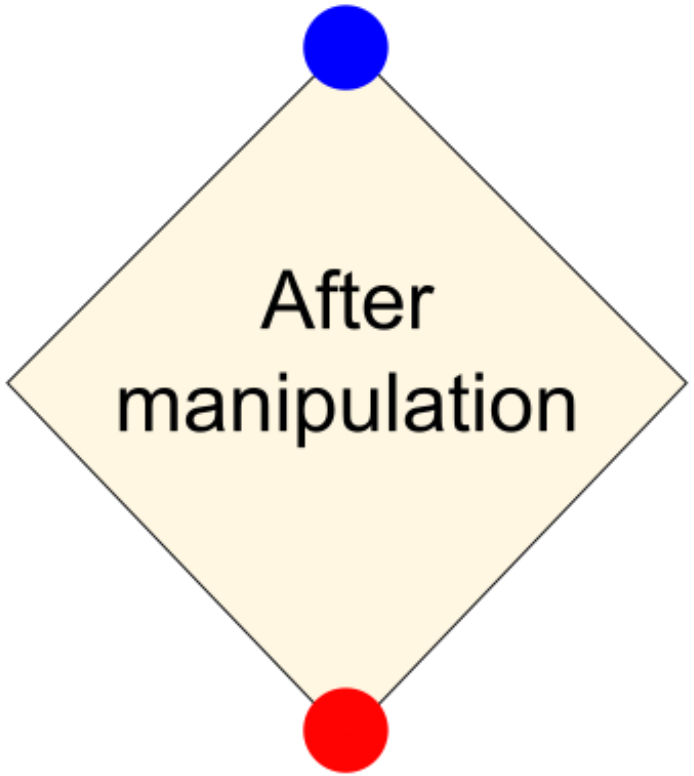

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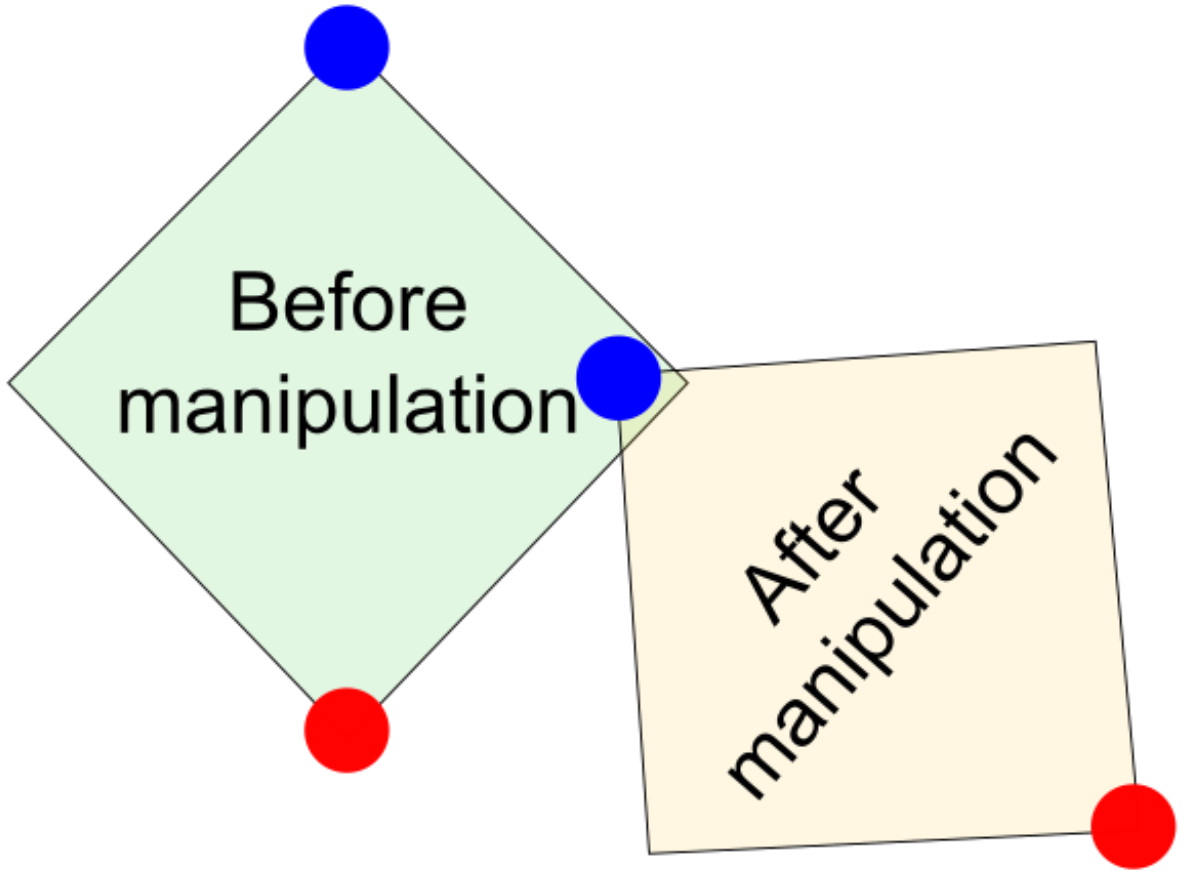

[Vaish and Garg, *IJCAI* 2017]

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

Stable marriages are manipulable,
but optimally manipulated marriages are stable.

DA fails strategyproofness---too bad!

Let's think of a different stable matching algorithm that is truthful.

[Roth, *MOR* 1982]

No stable matching procedure can be strategyproof.

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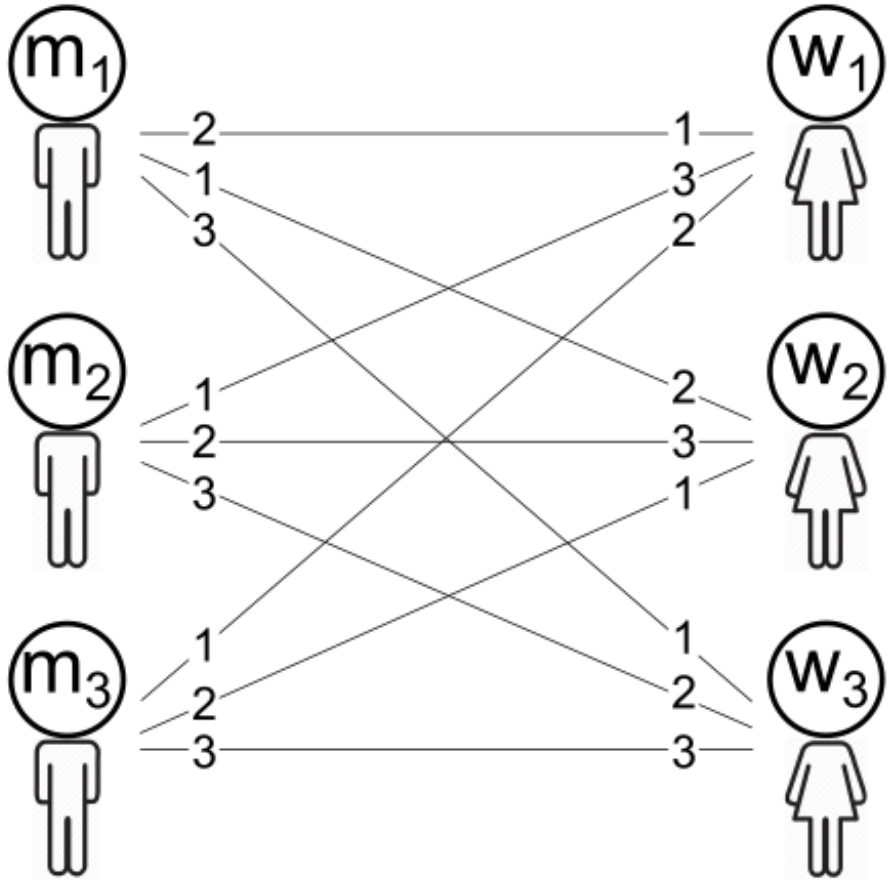

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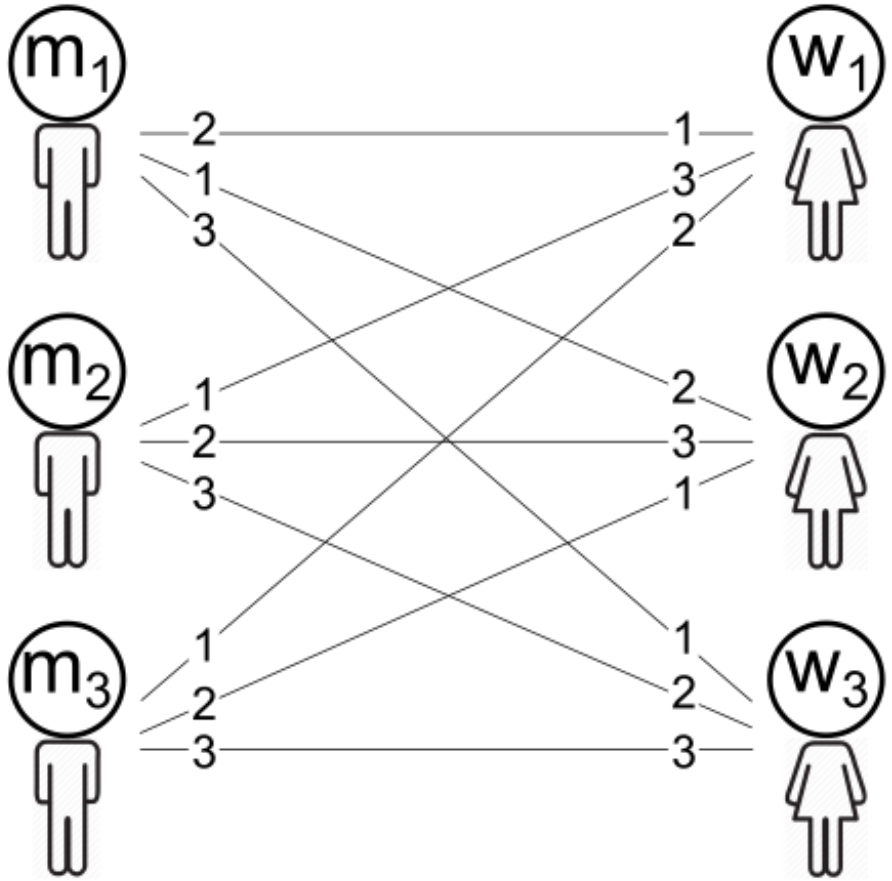

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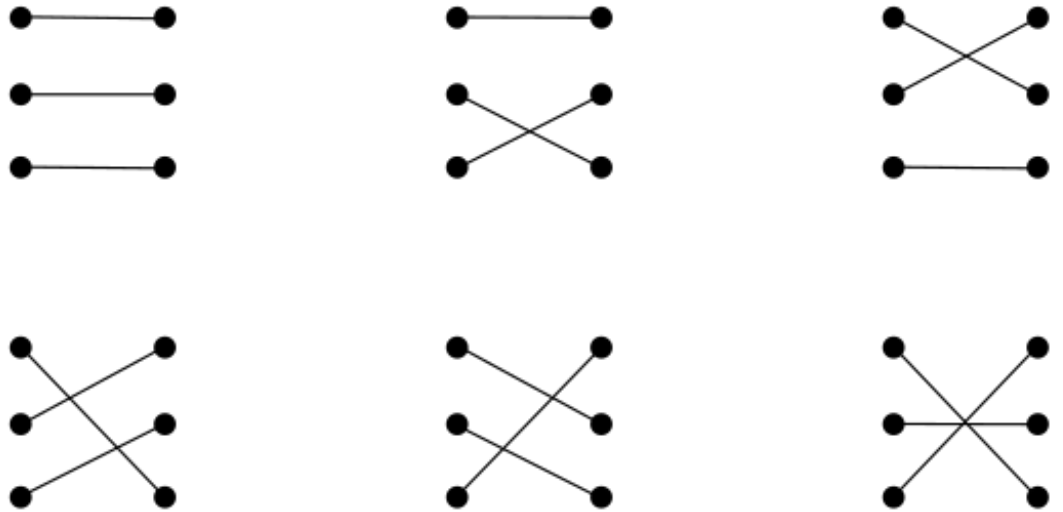

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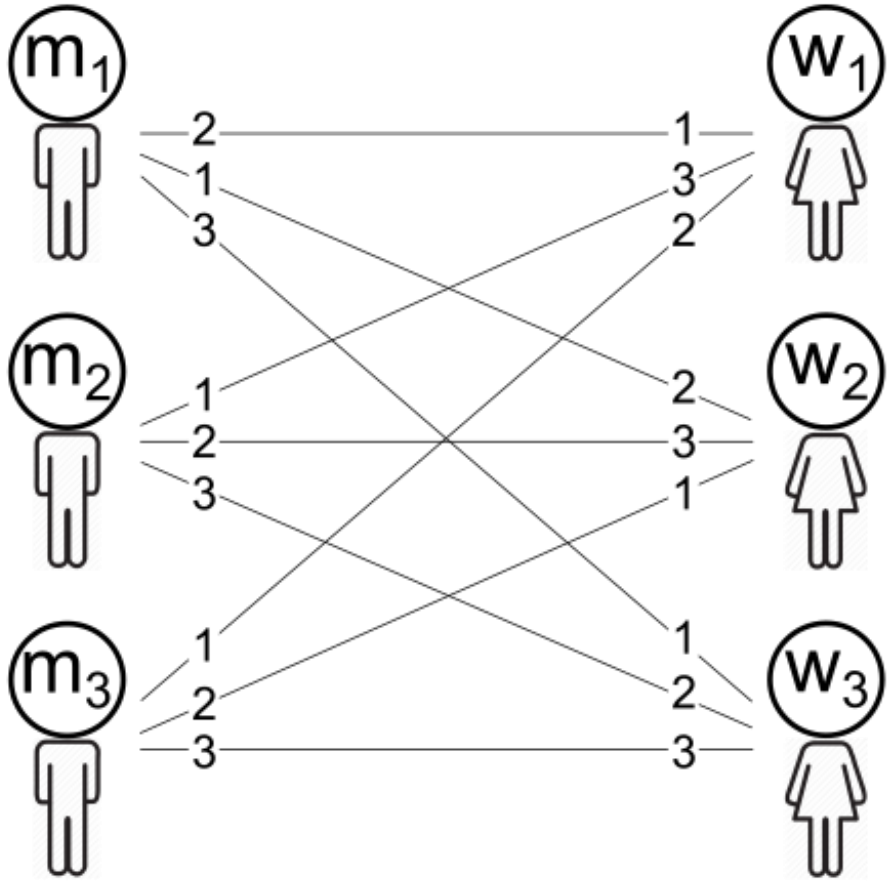

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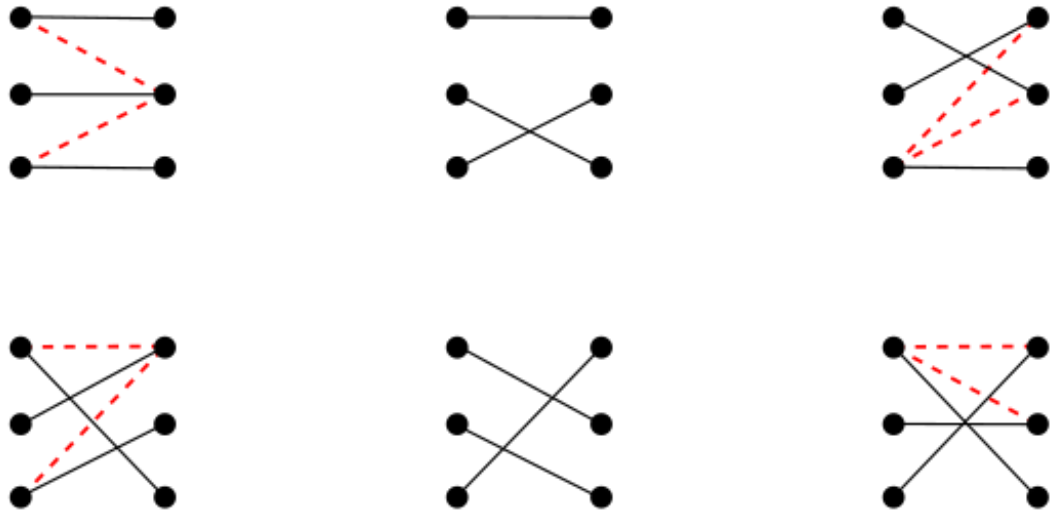

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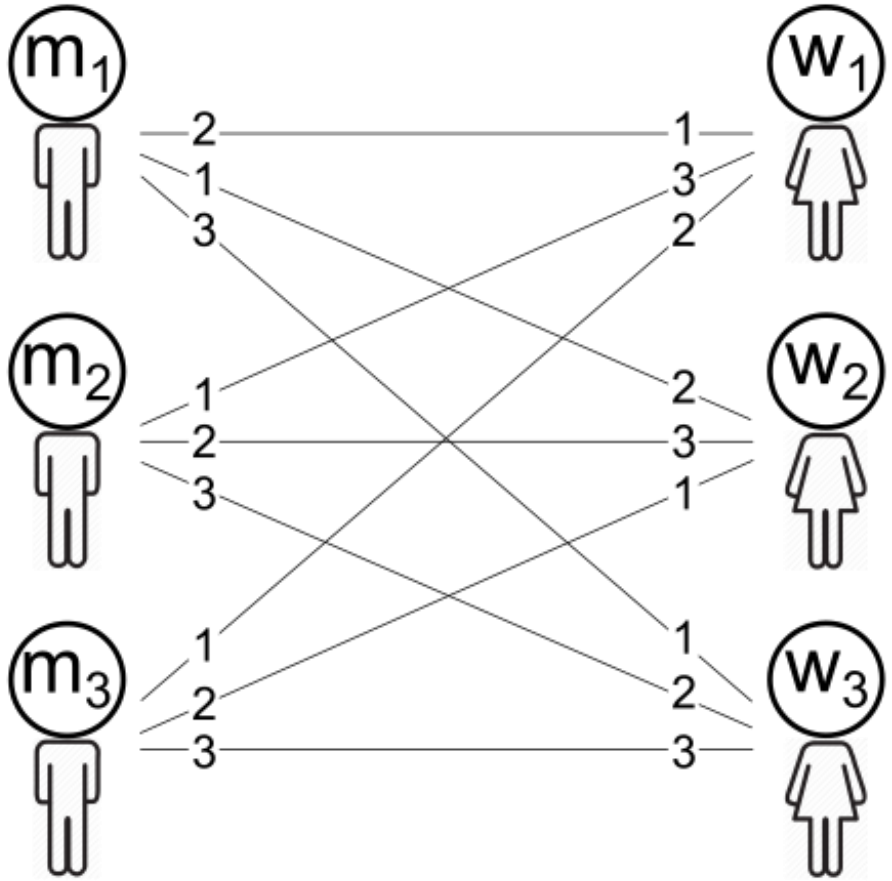

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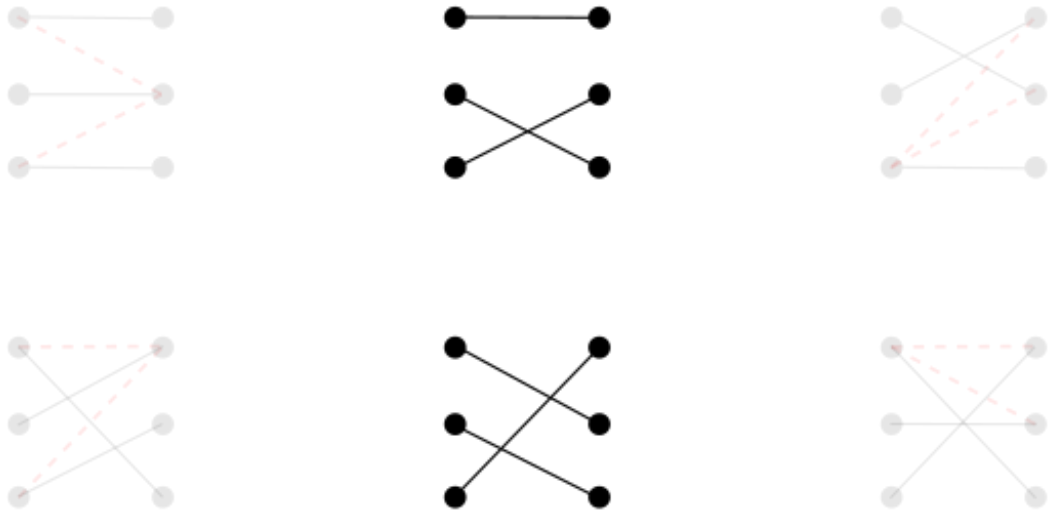

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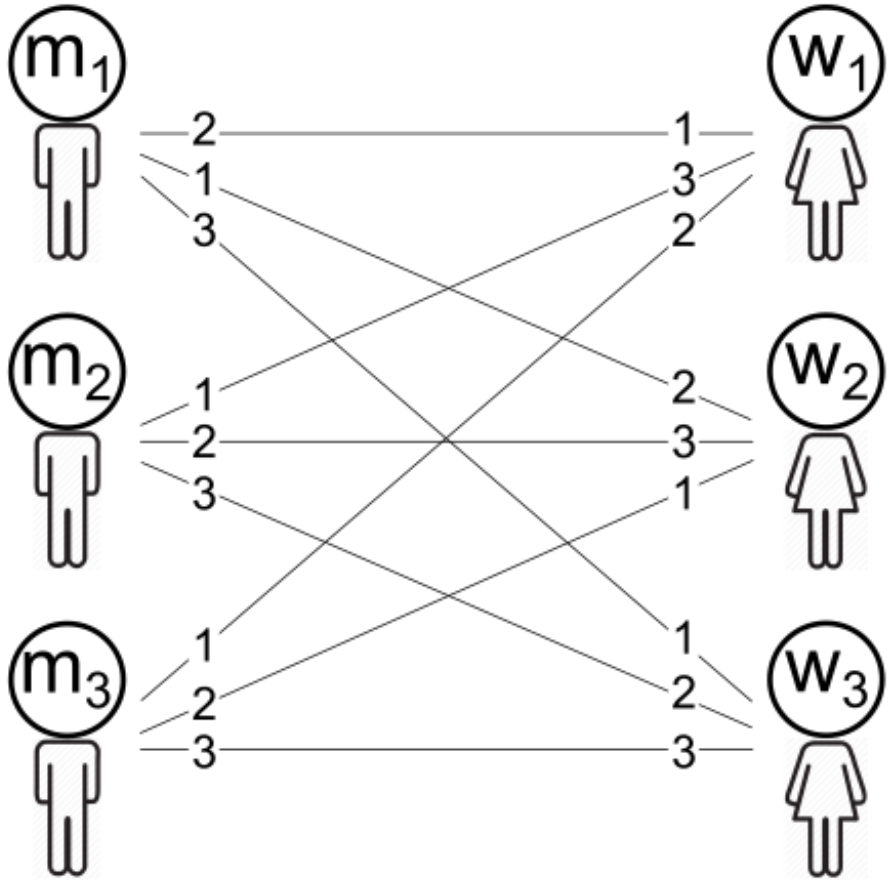

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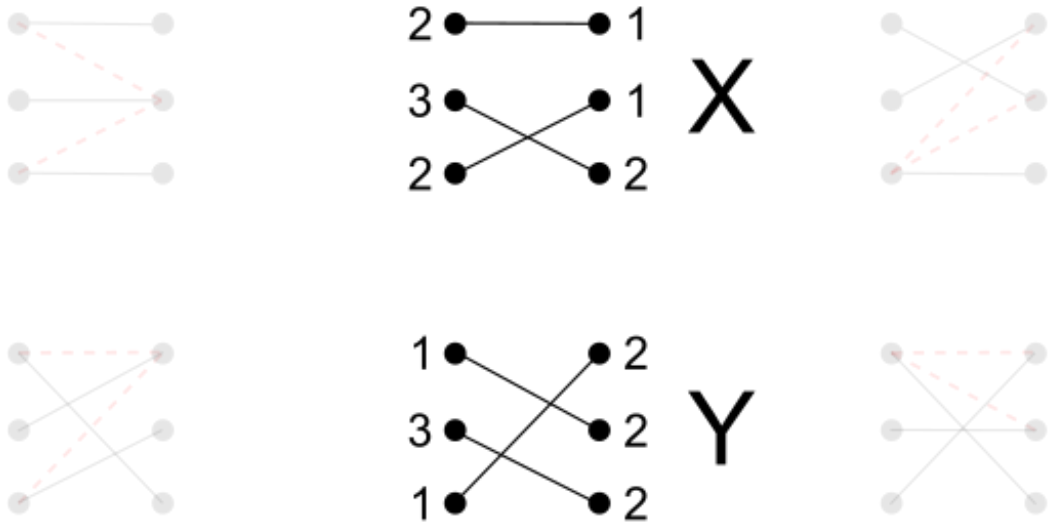

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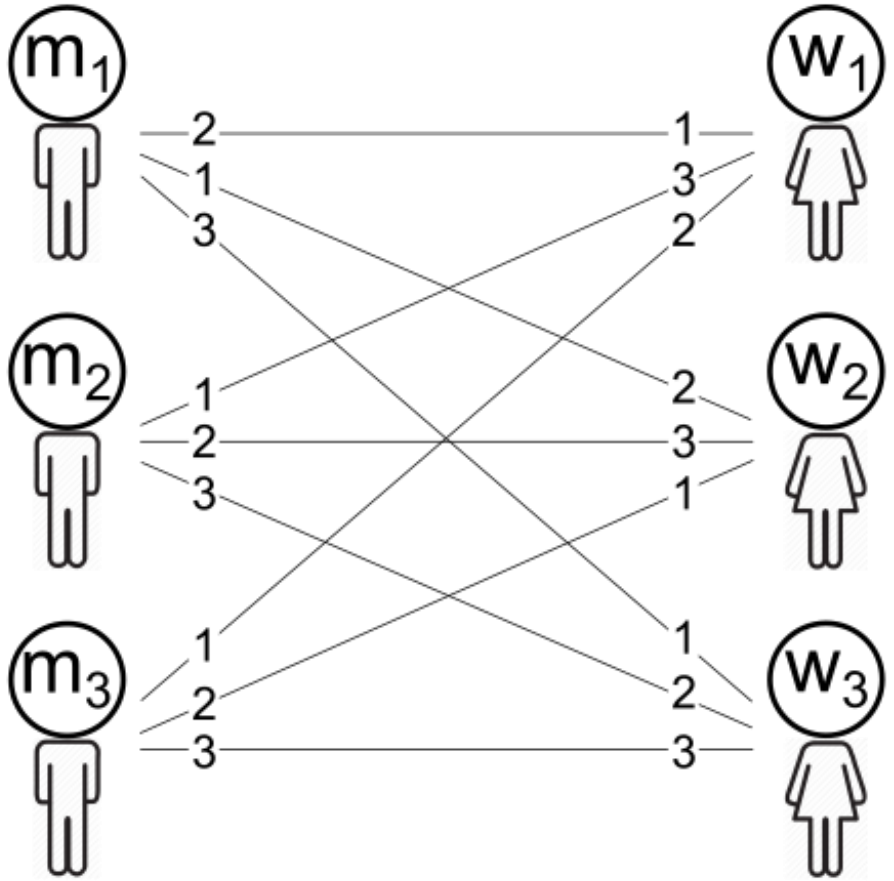

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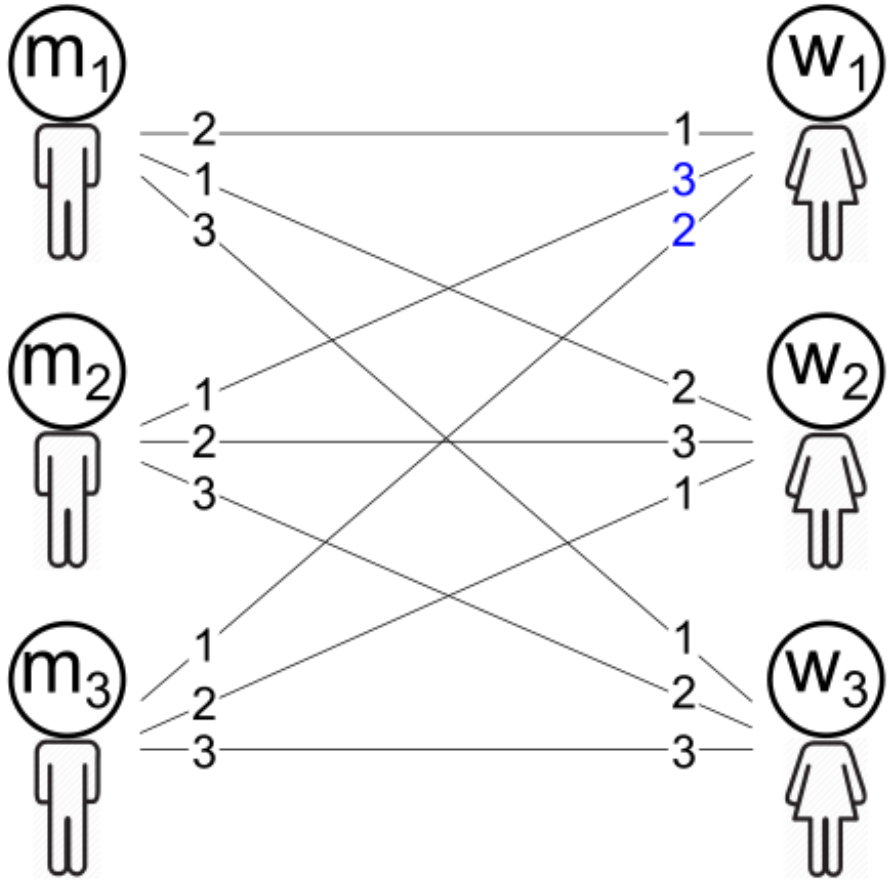

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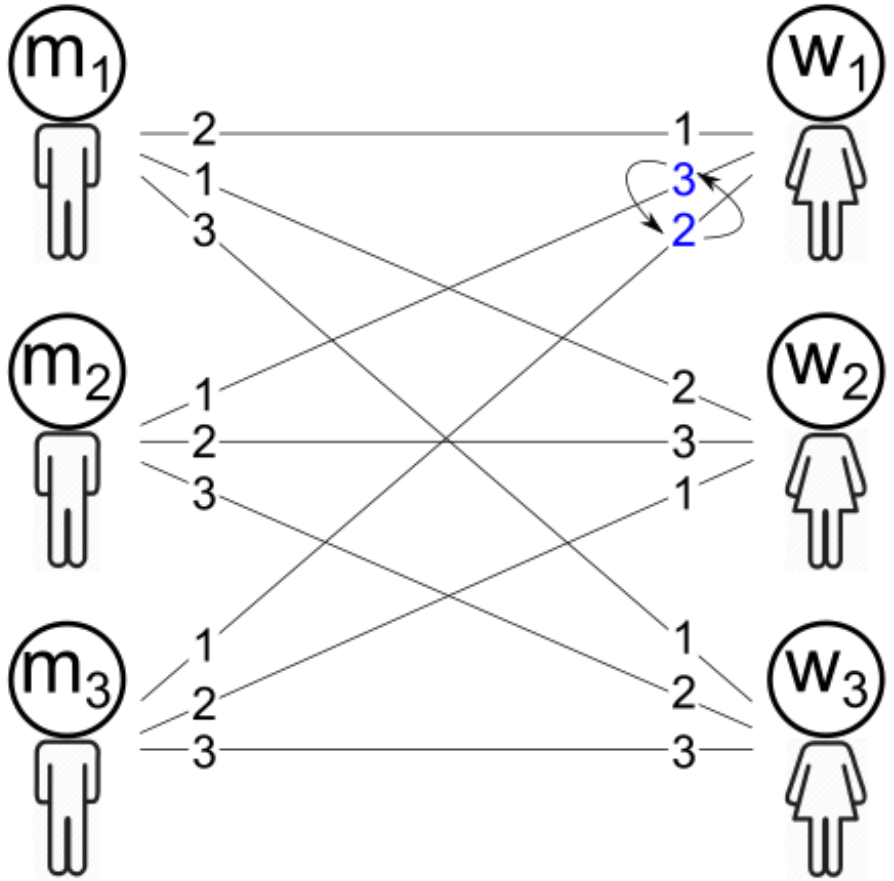

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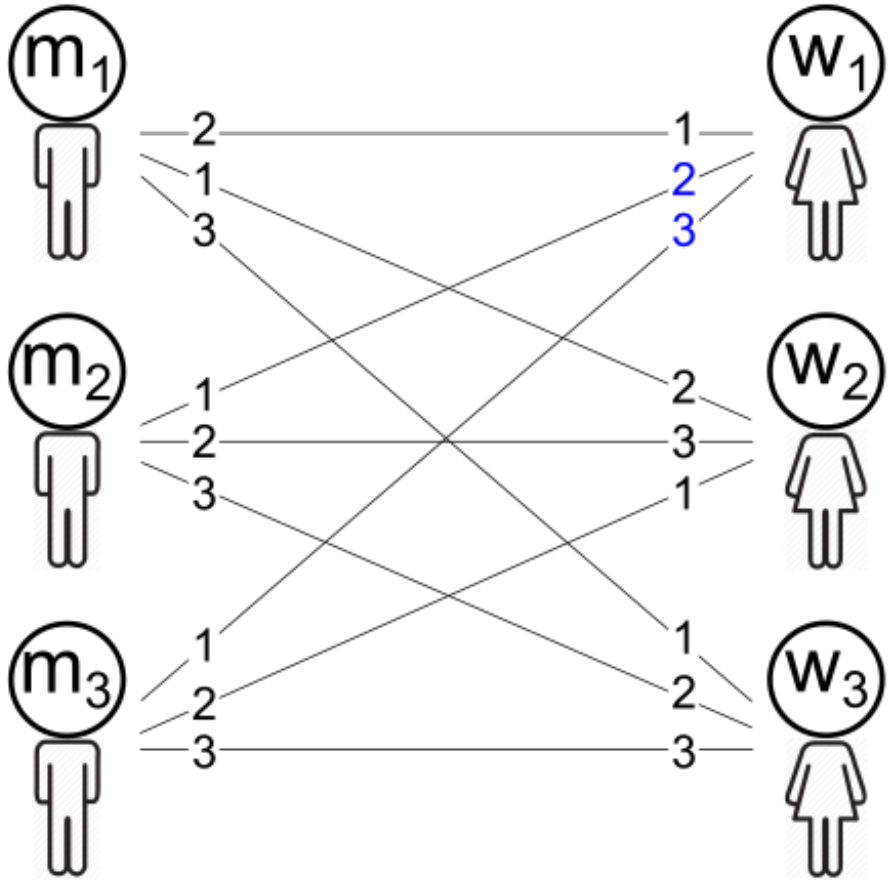

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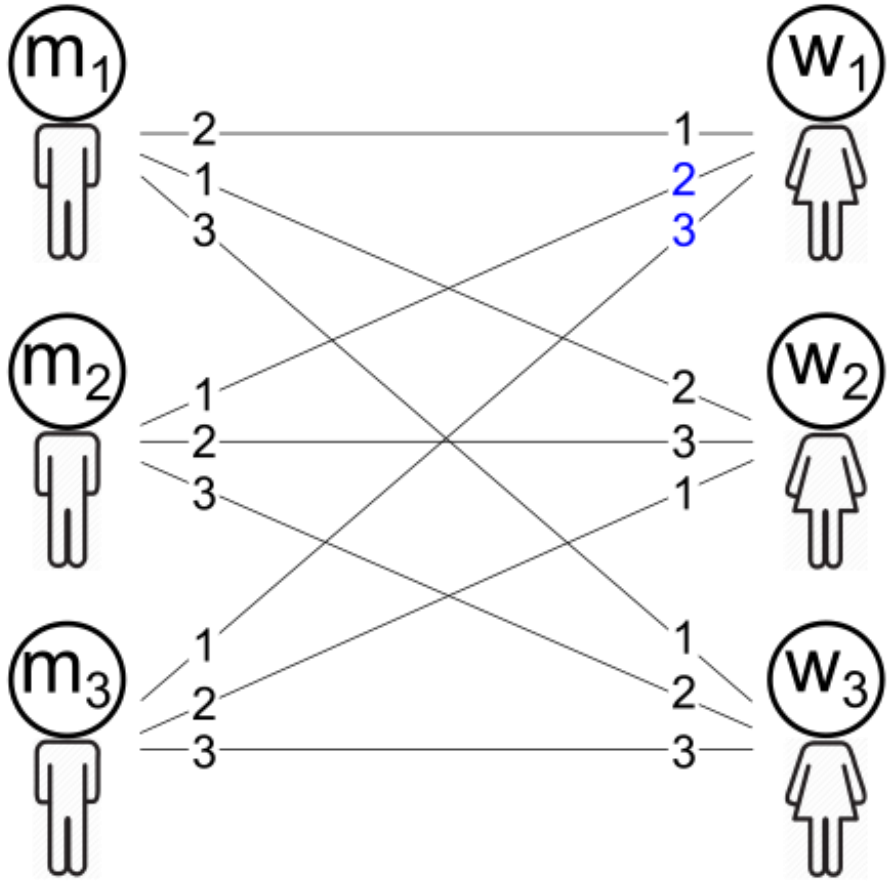

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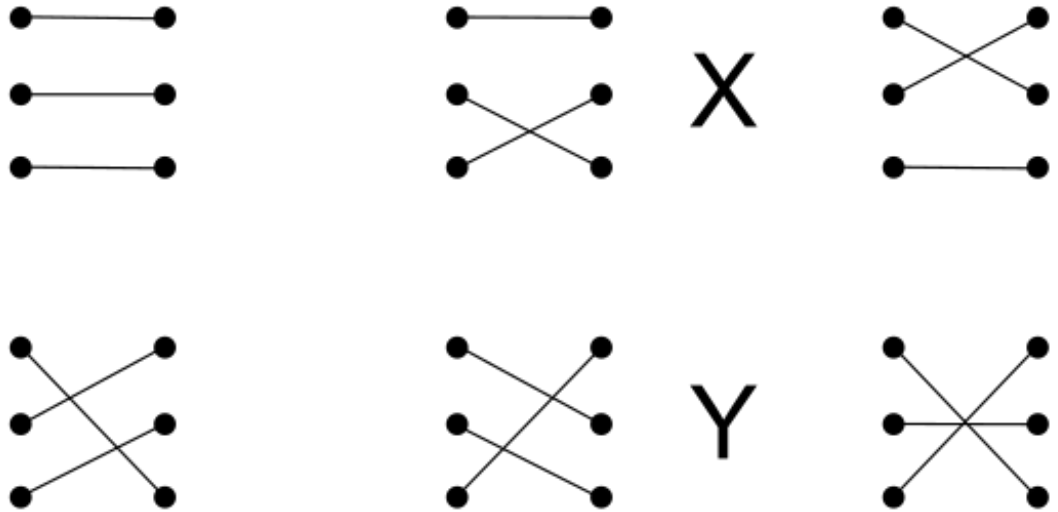

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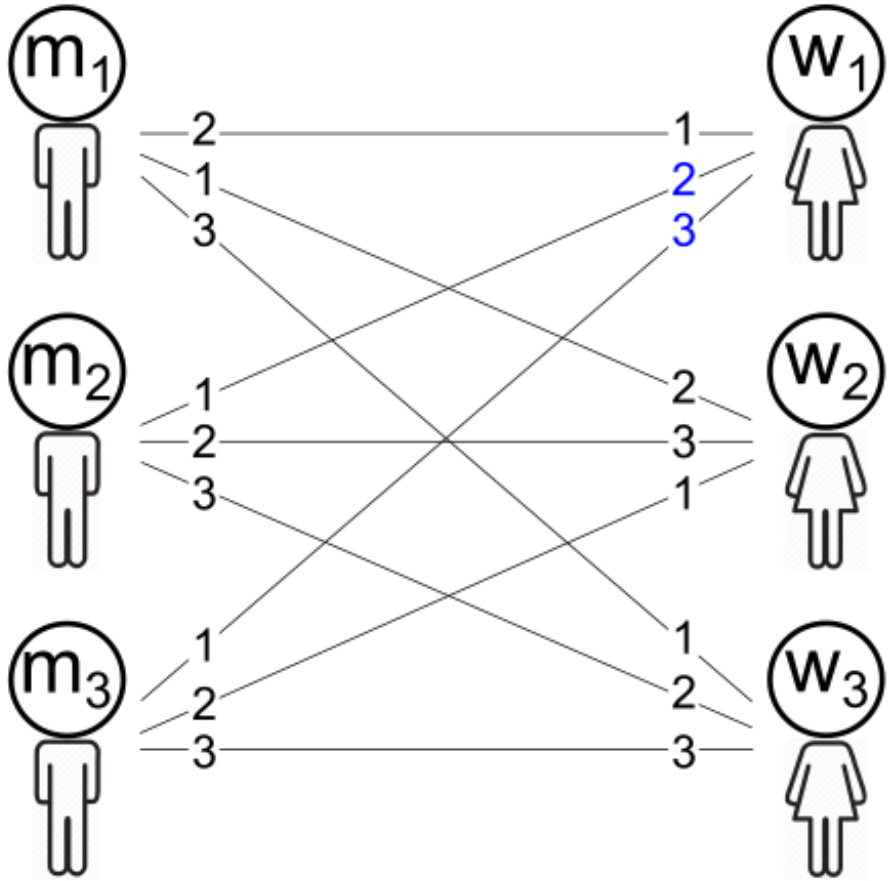

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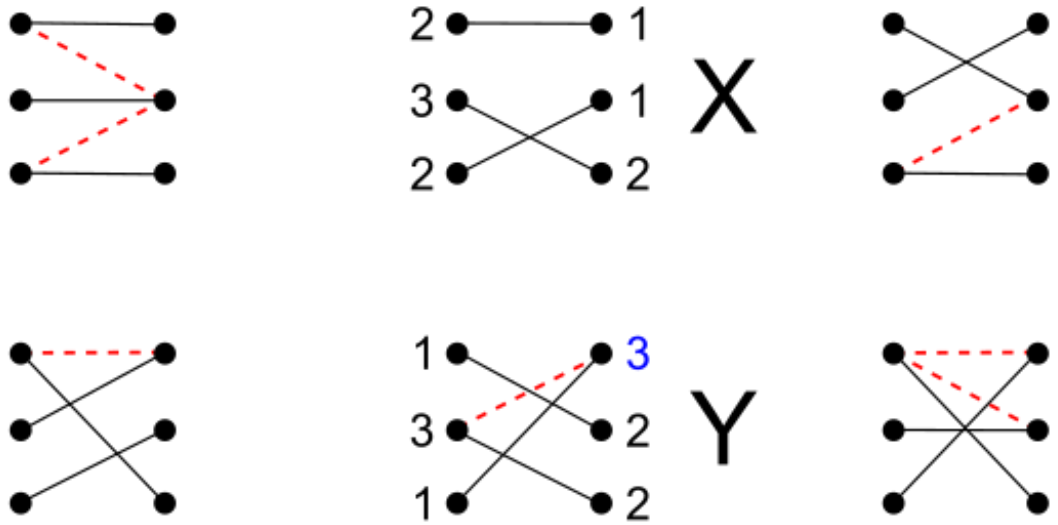

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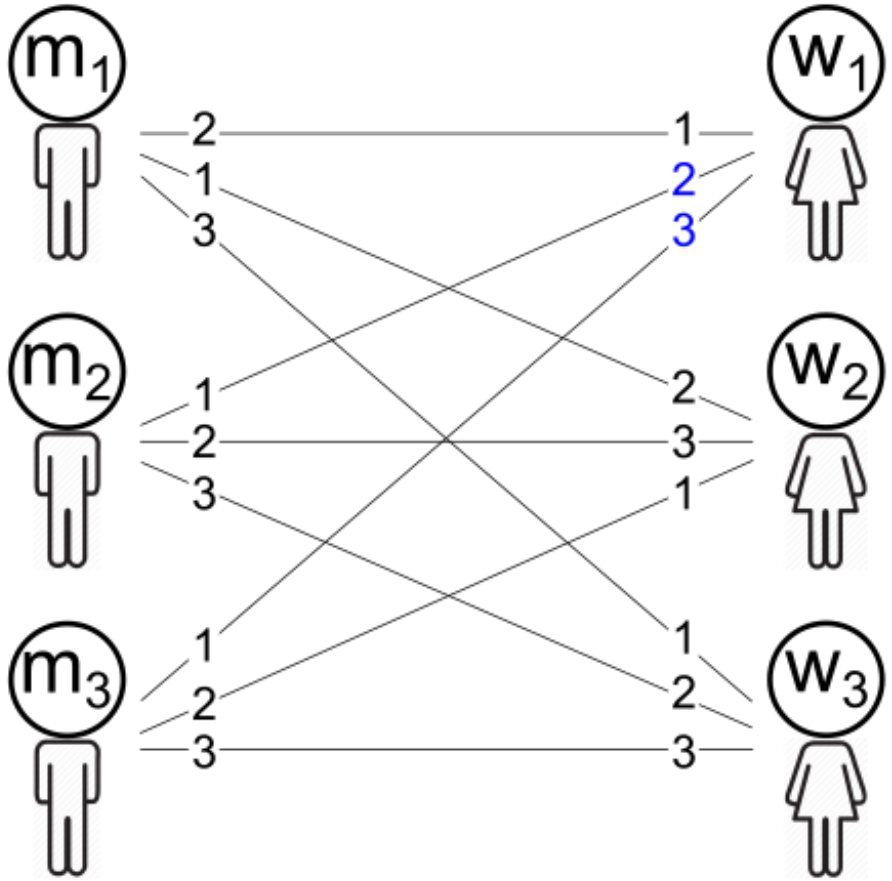

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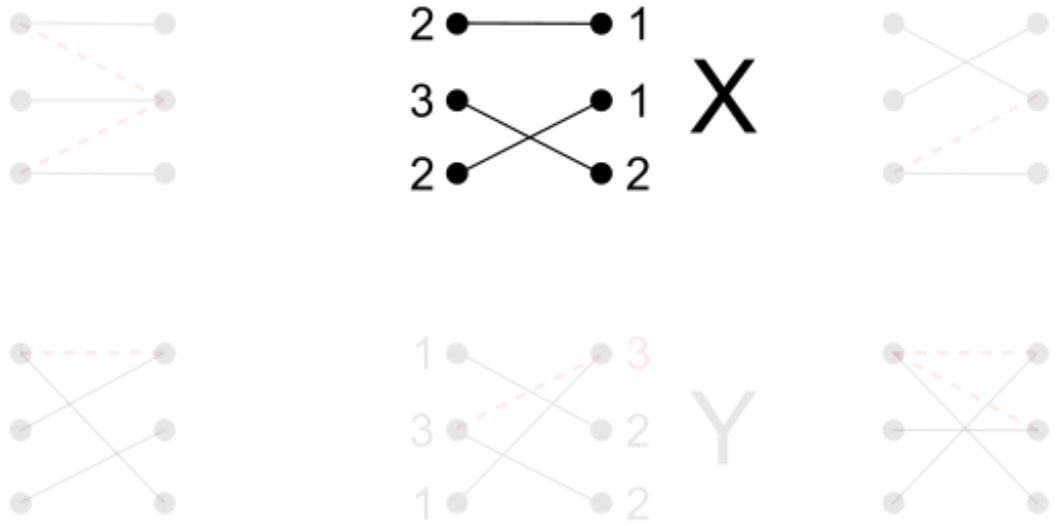

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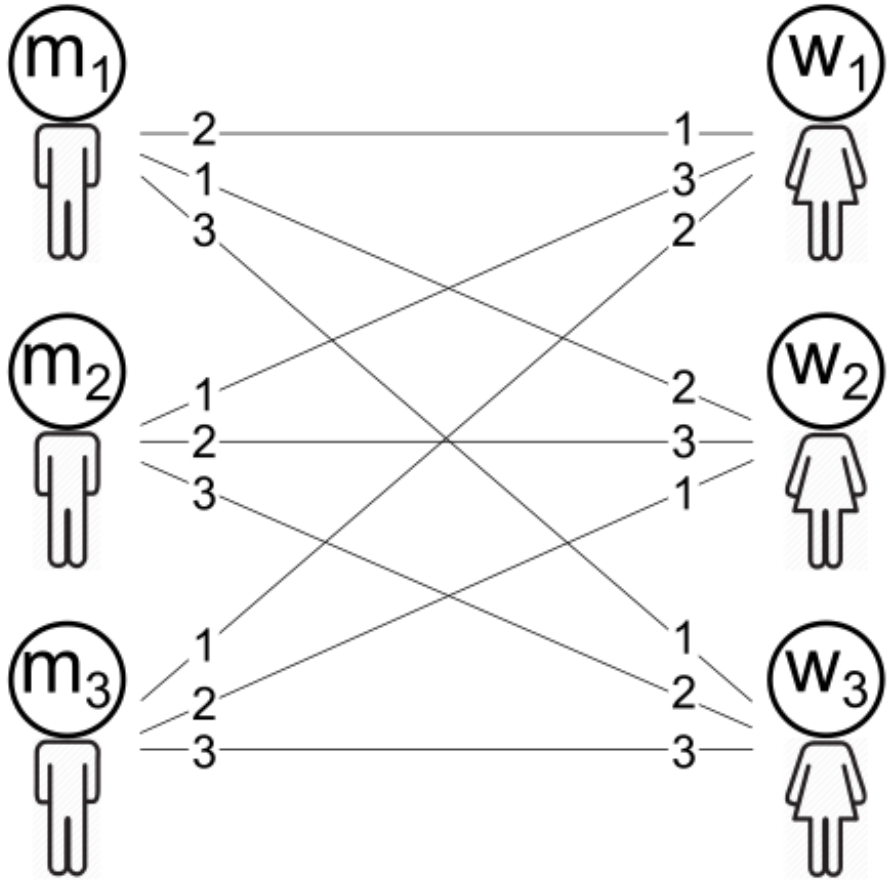

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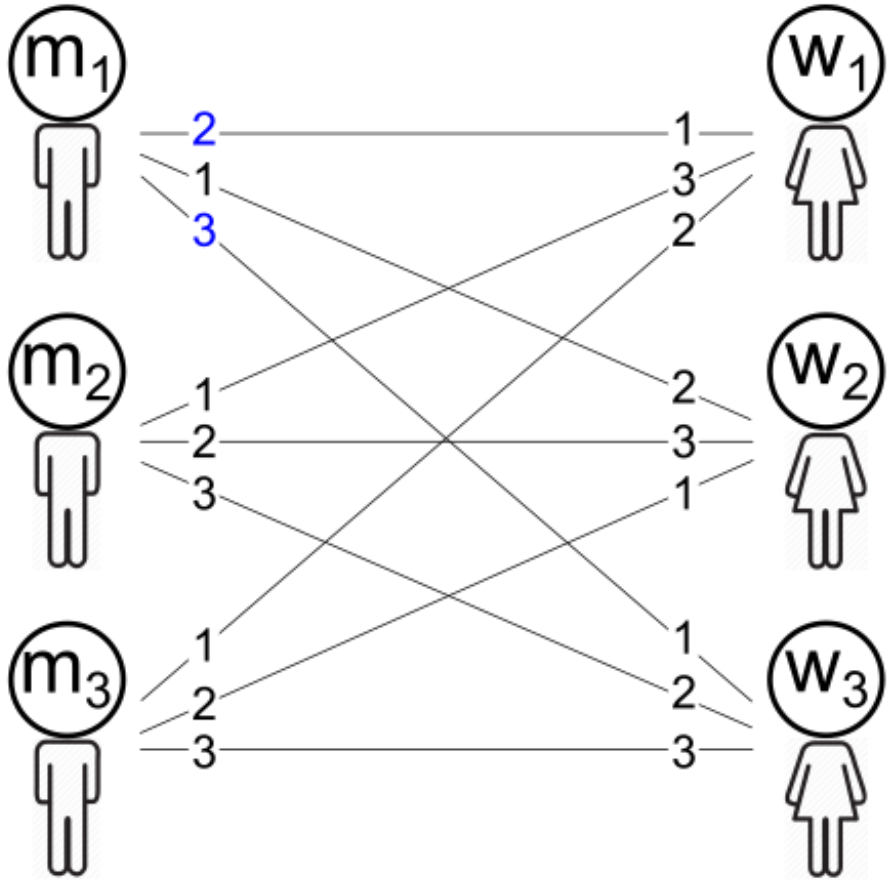

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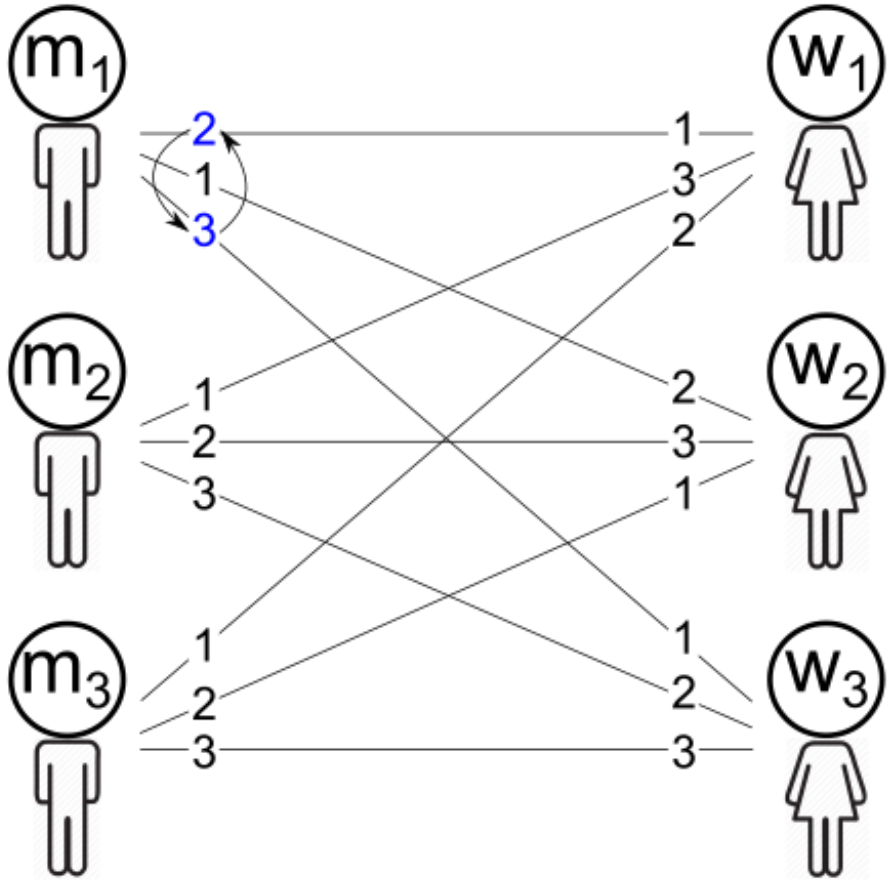

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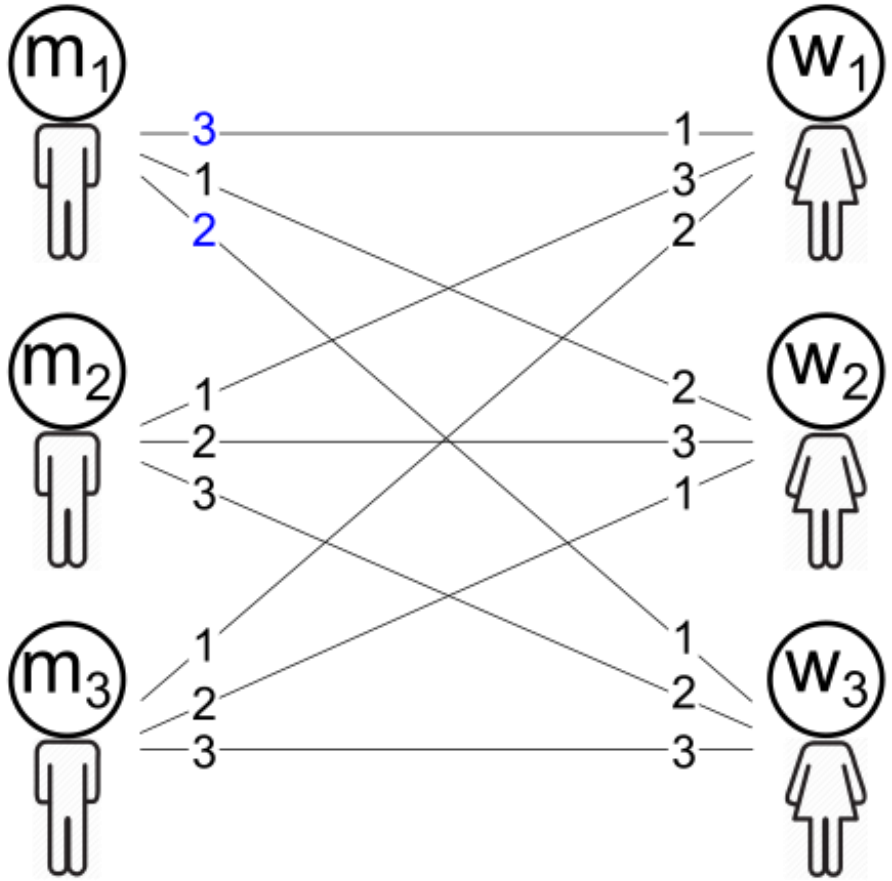

Instance I_2

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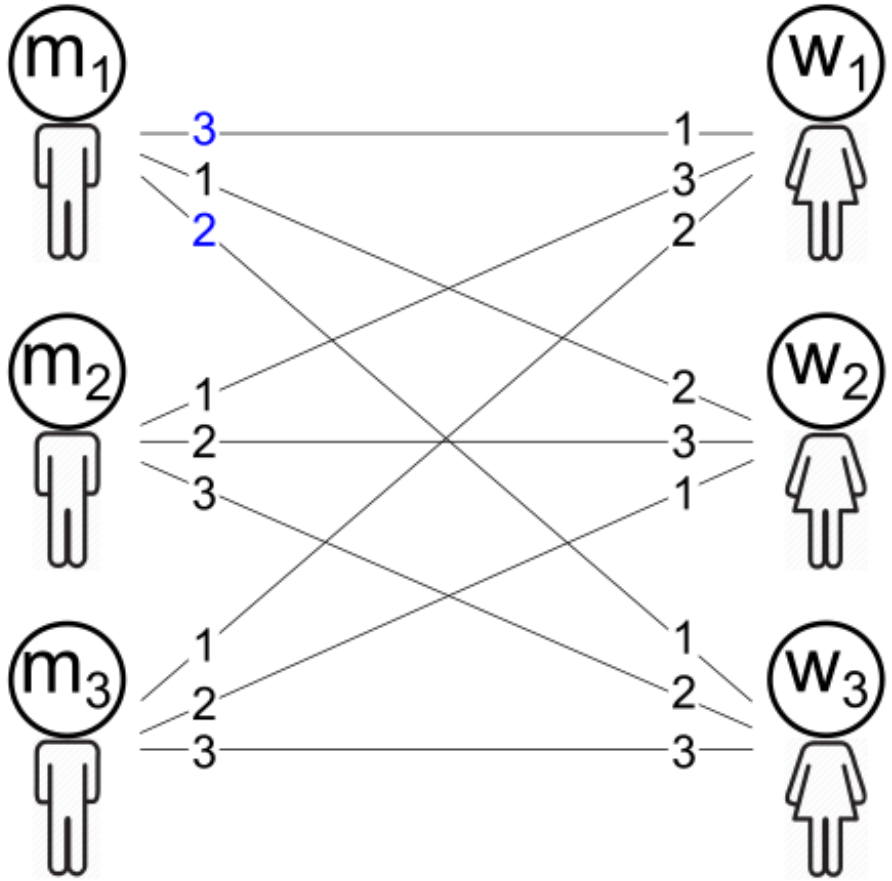

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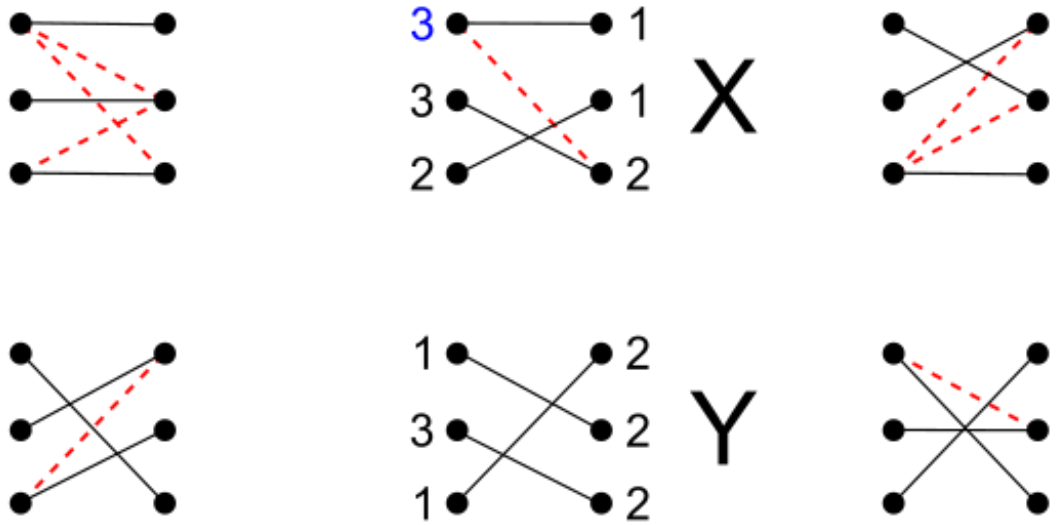

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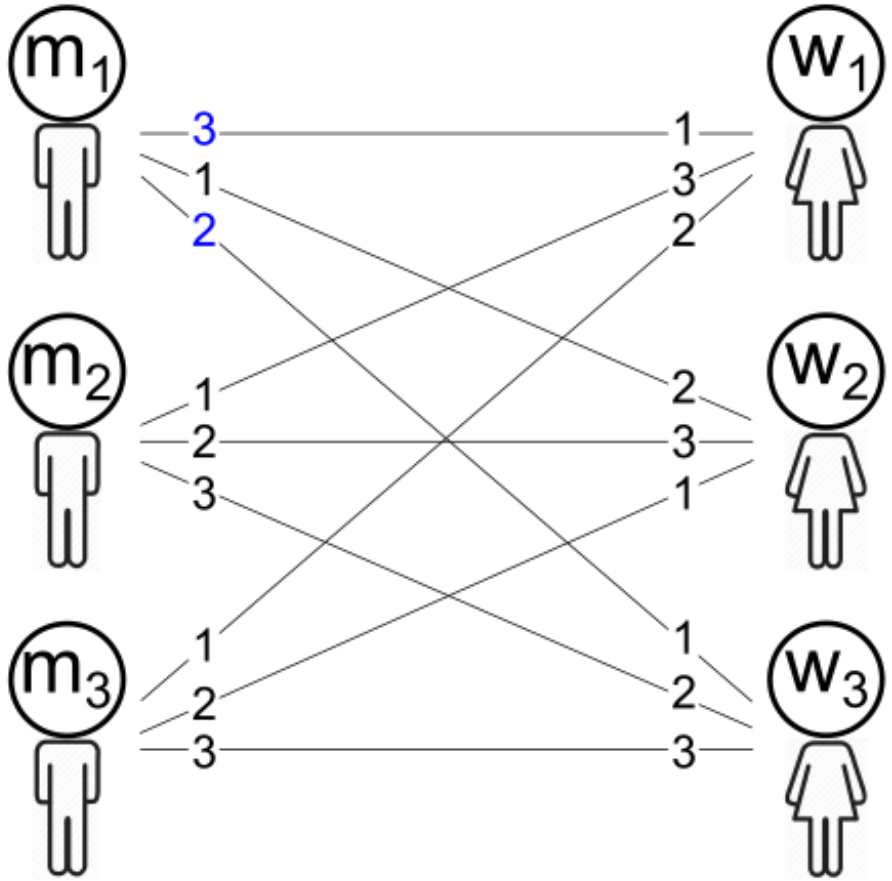

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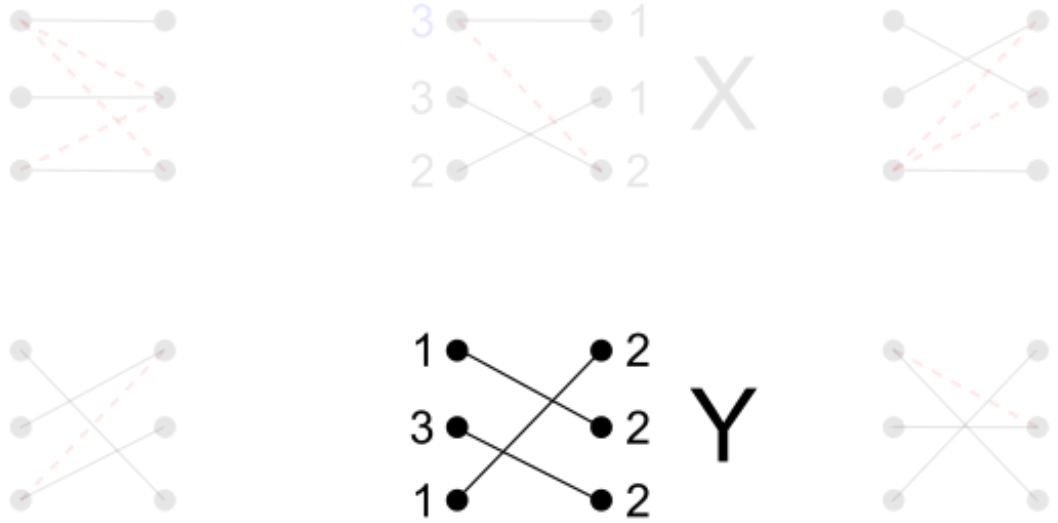

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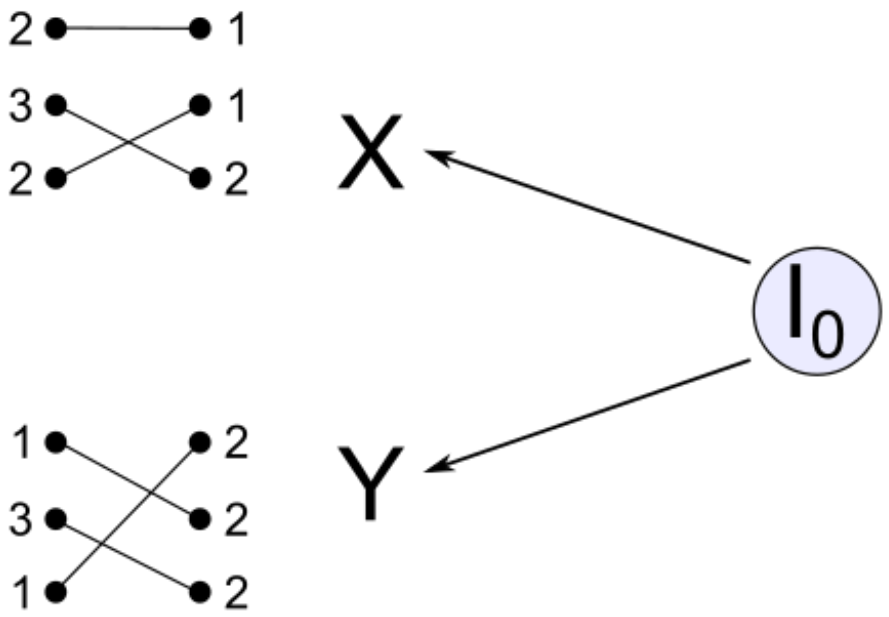

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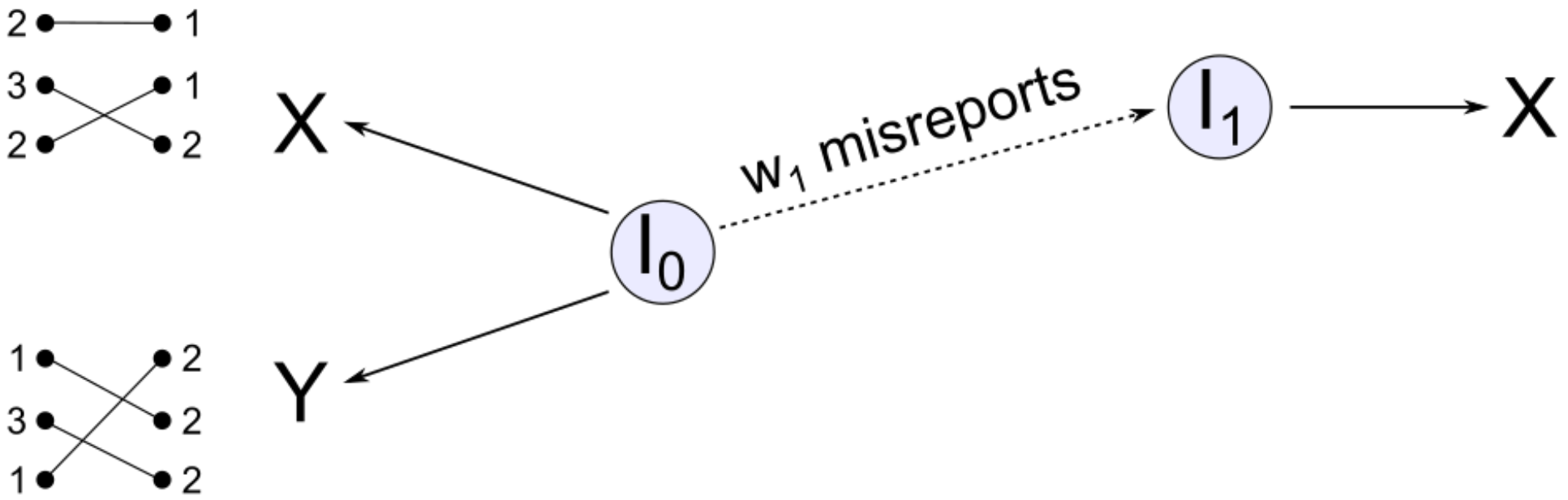

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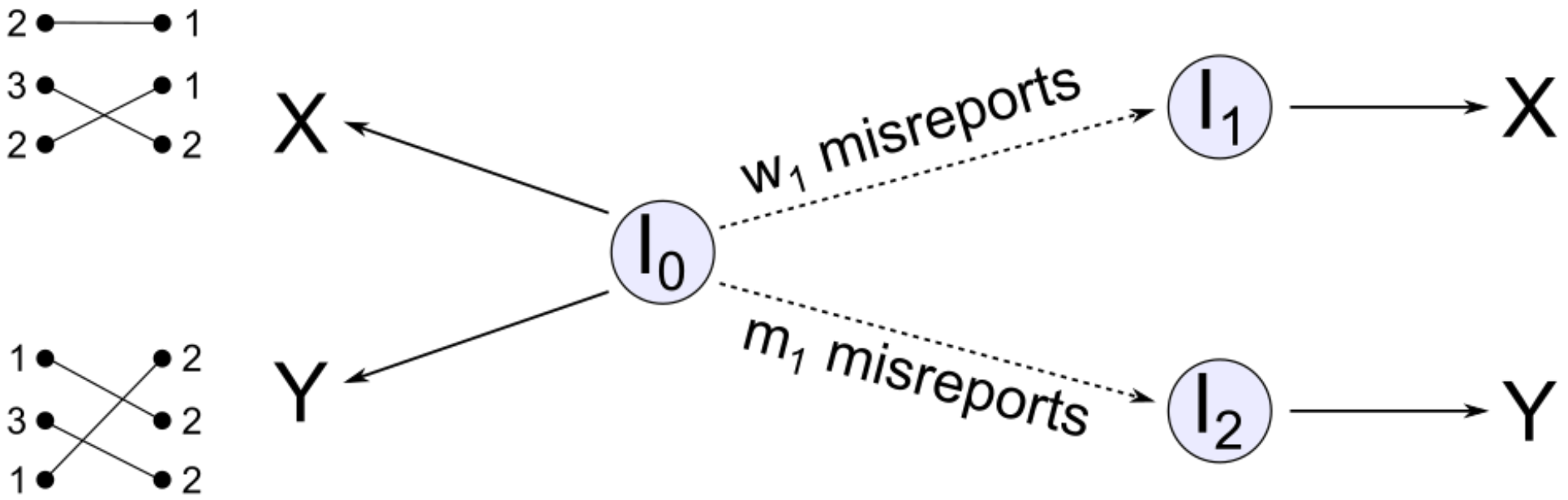

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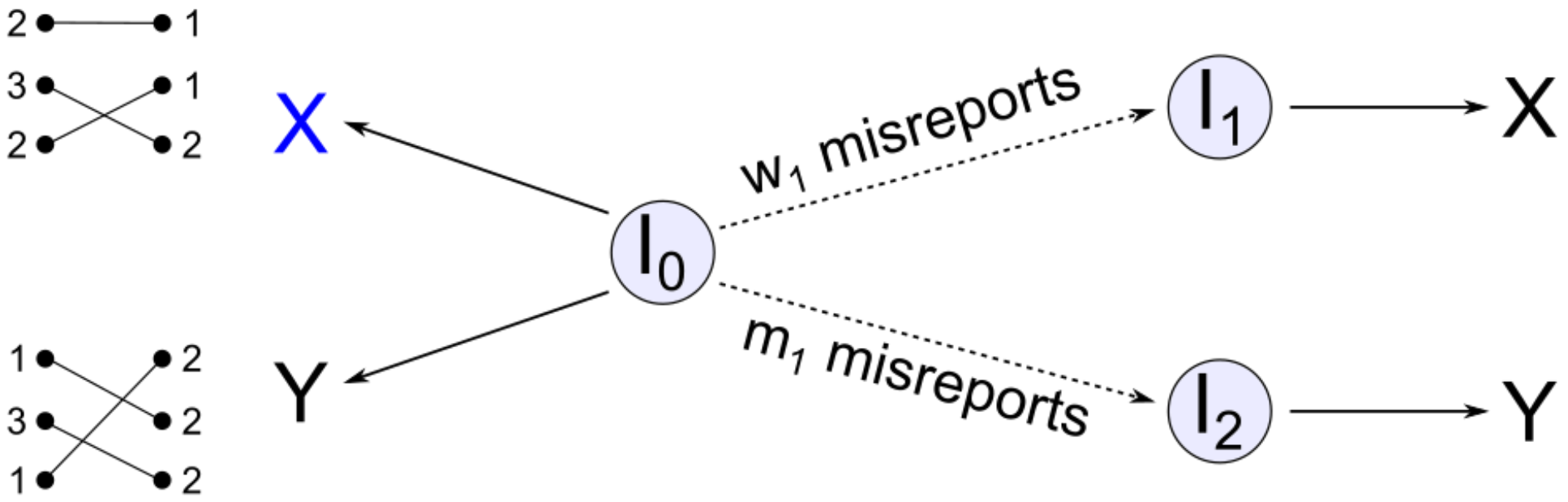

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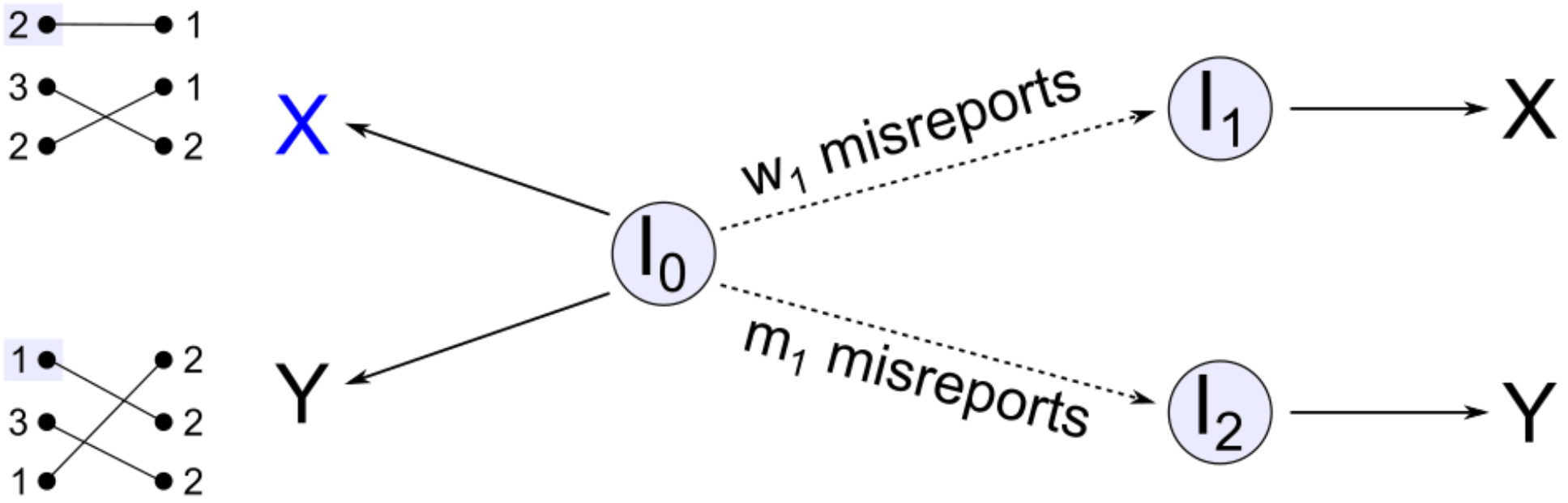

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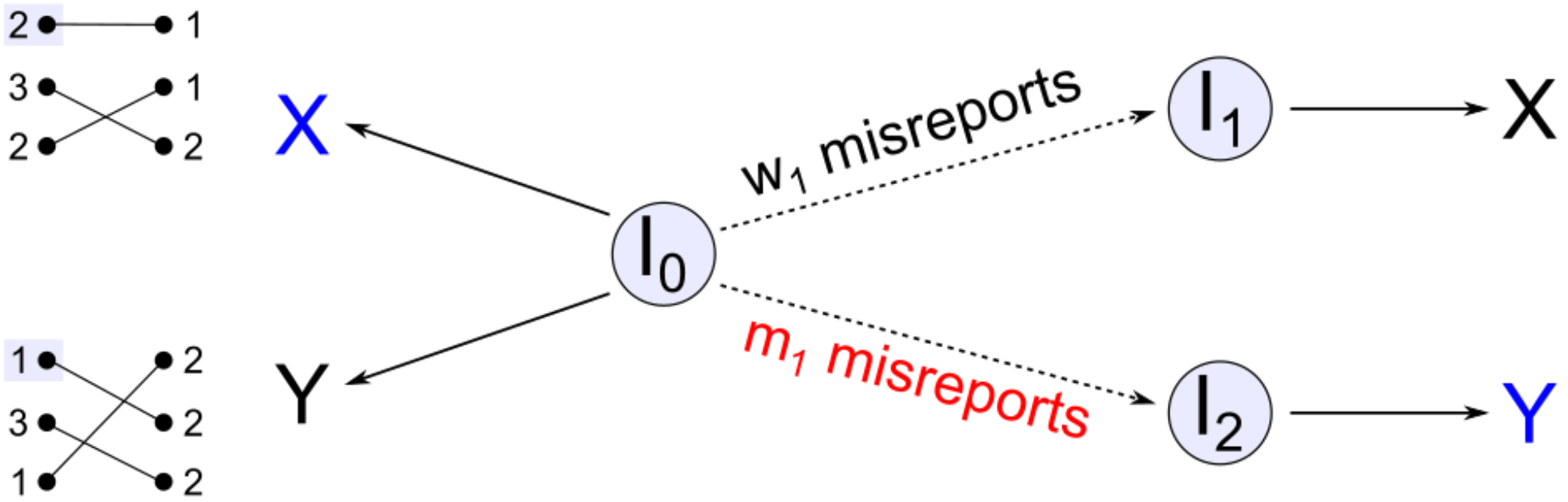

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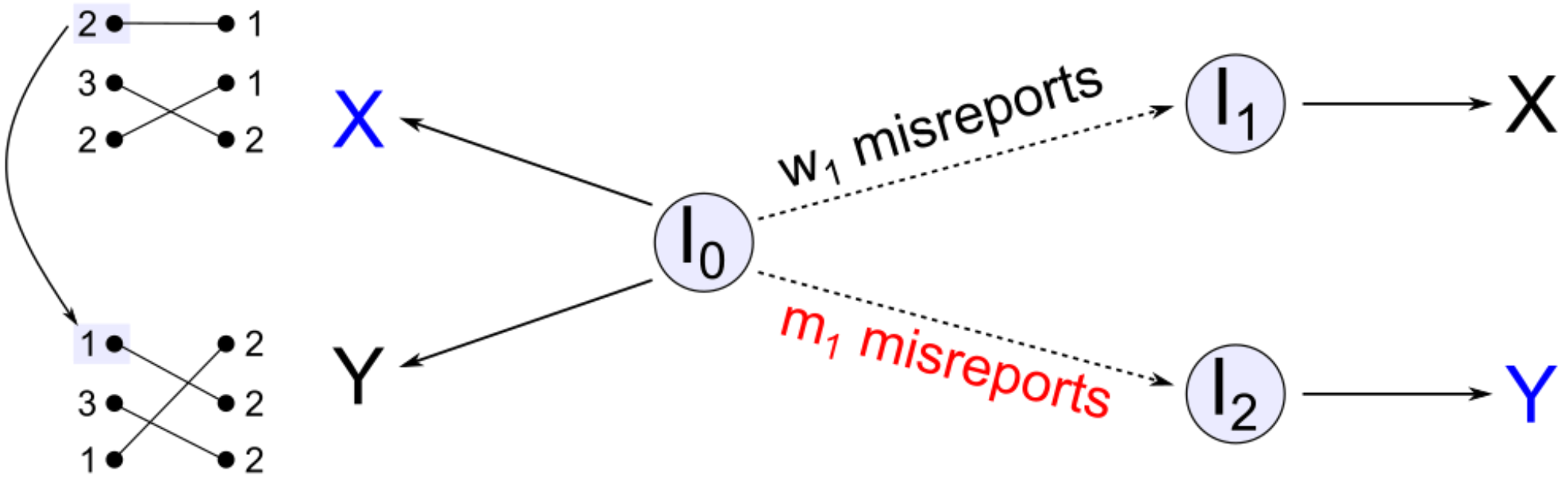

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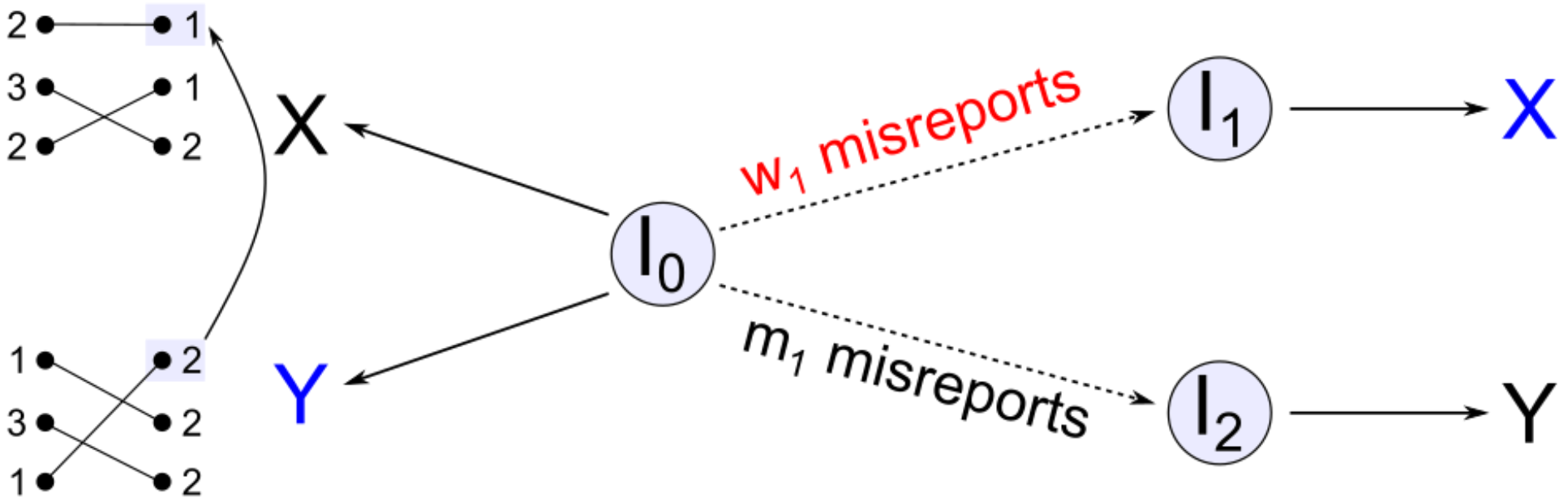

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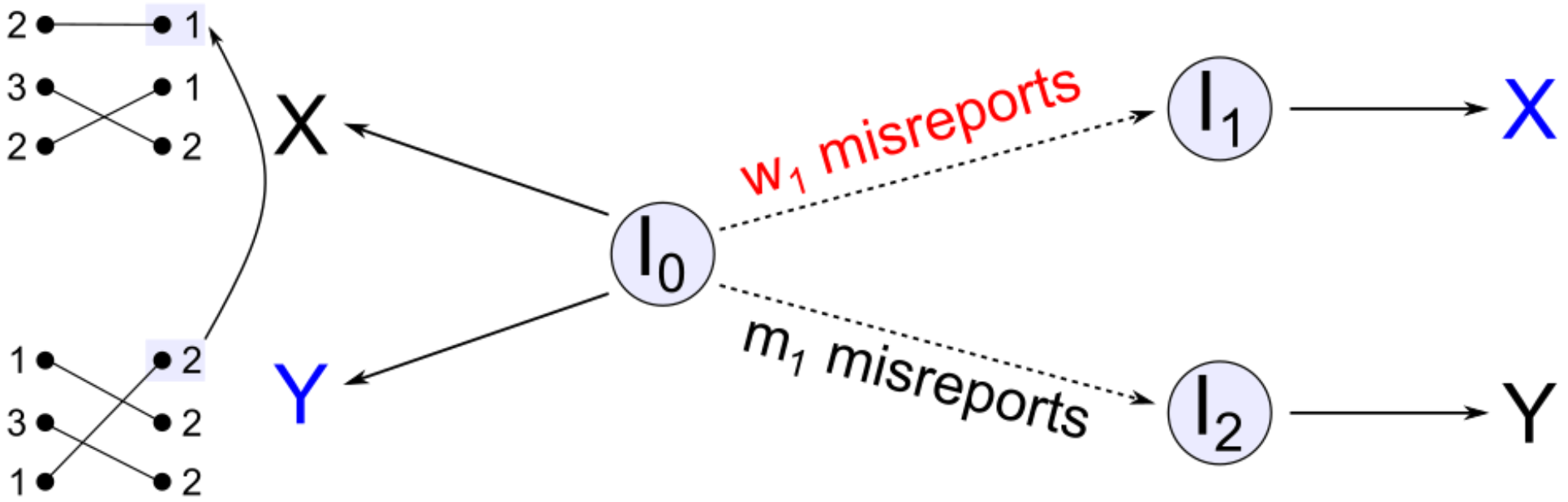

RECAP

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No stable matching procedure is strategyproof for all agents.

Next Time

Finding Fair Stable Matchings

References

- DA algorithm fails to be strategyproof.

  Lester Dubins and David Freedman
  "*Machiavelli and the Gale-Shapley Algorithm*"
  American Mathematical Monthly, 88(7), 1981 pg 485-494
  https://www.jstor.org/stable/2321753

- No stable matching procedure is strategyproof.

  Alvin E Roth
  "*The Economics of Matching: Stability and Incentives*"
  Mathematics of Operations Research, 7(4), 1982 pg 617-628
  https://pubsonline.informs.org/doi/abs/10.1287/moor.7.4.617

References

- An algorithm for finding an optimal manipulation for a woman.

  Chung-Piaw Teo, Jay Sethuraman, and Wee-Peng Tan
  "*Gale-Shapley Stable Marriage Problem Revisited: Strategic Issues and Applications*"
  Management Science, 47(9), 2001 pg 252–1267
  https://doi.org/10.1287/mnsc.47.9.1252.9784

- Optimally manipulated marriages are stable.

  Rohit Vaish and Dinesh Garg
  "*Manipulating Gale-Shapley Algorithm: Preserving Stability and Remaining Inconspicuous*"
  IJCAI 2017, pg 437-443
  https://www.ijcai.org/proceedings/2017/62

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

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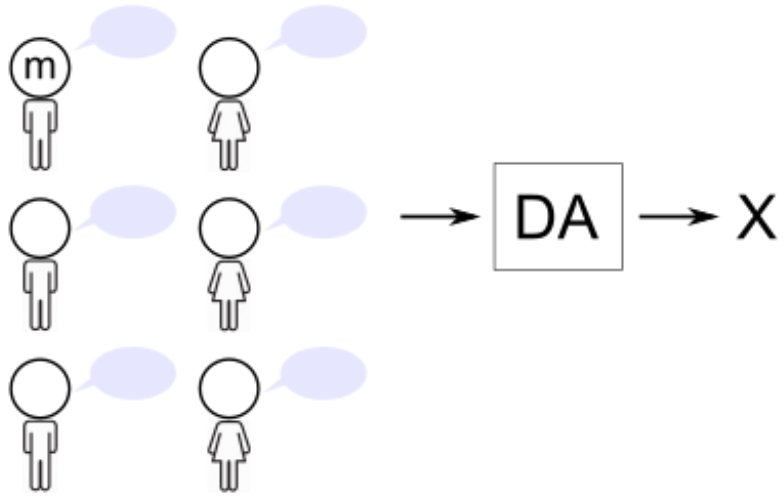

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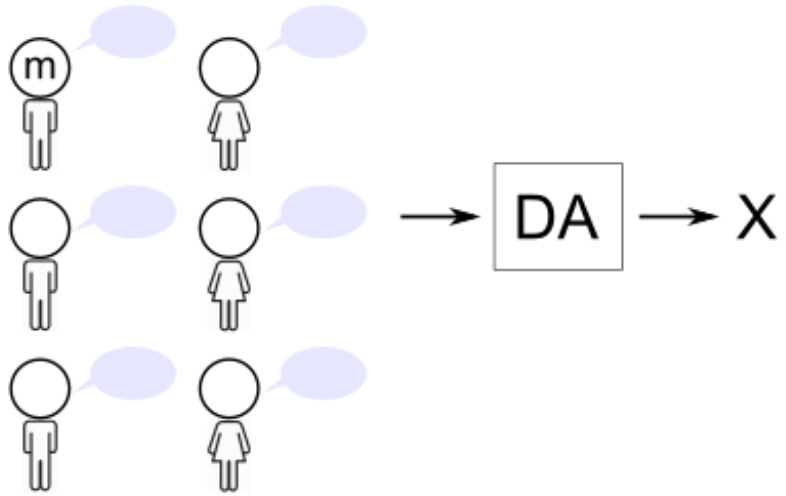

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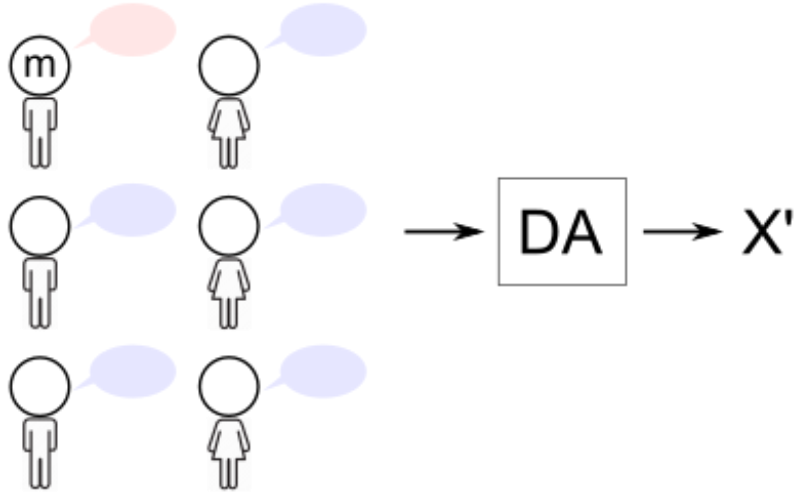

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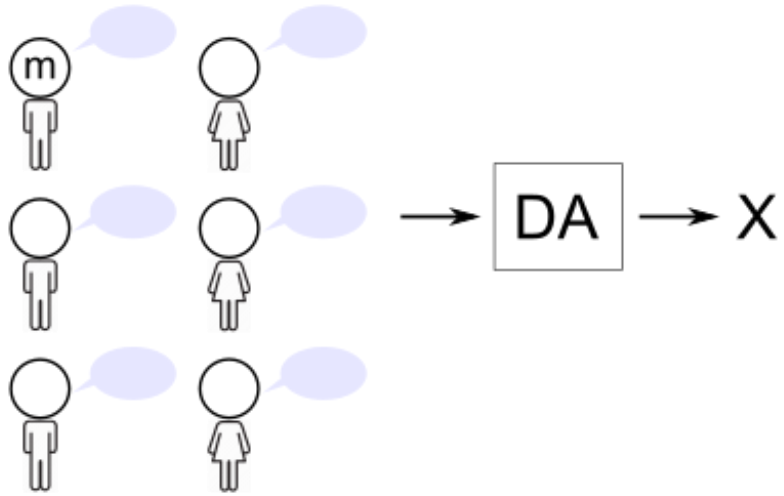

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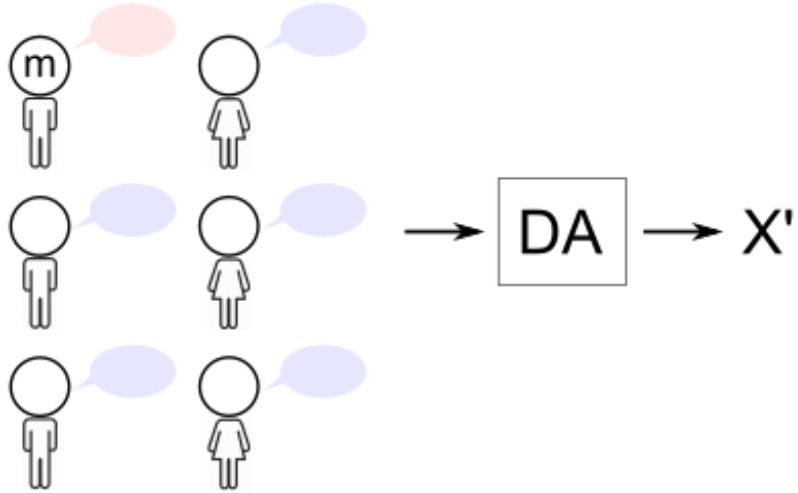

P_m ... > X'(m) > ... > X(m) > ...

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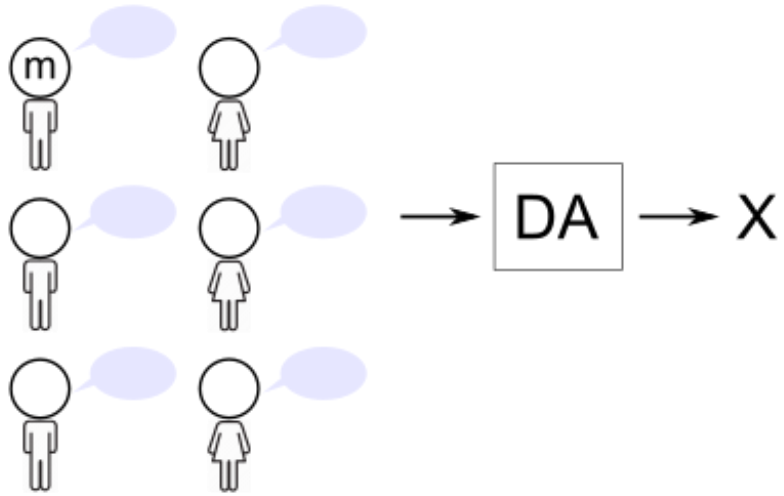

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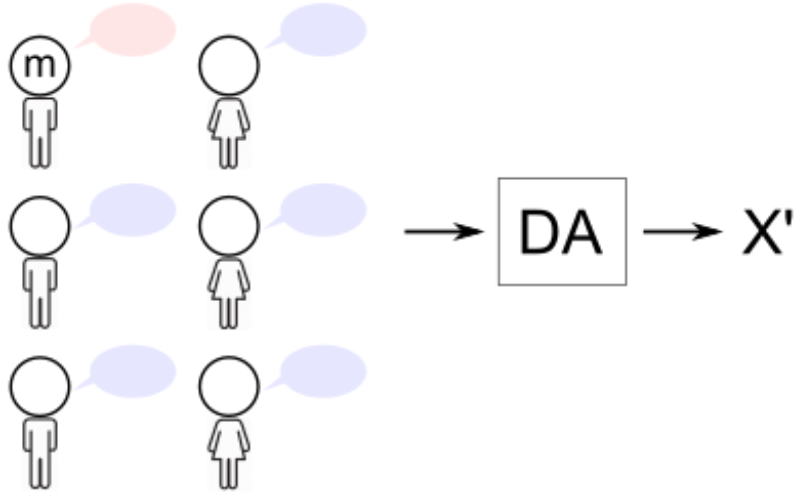

P_m ... > X'(m) > ... > X(m) > ...

We will use three lemmas to derive a contradiction.

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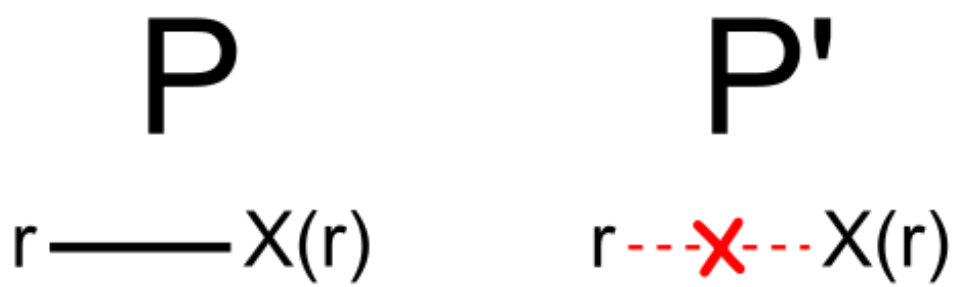

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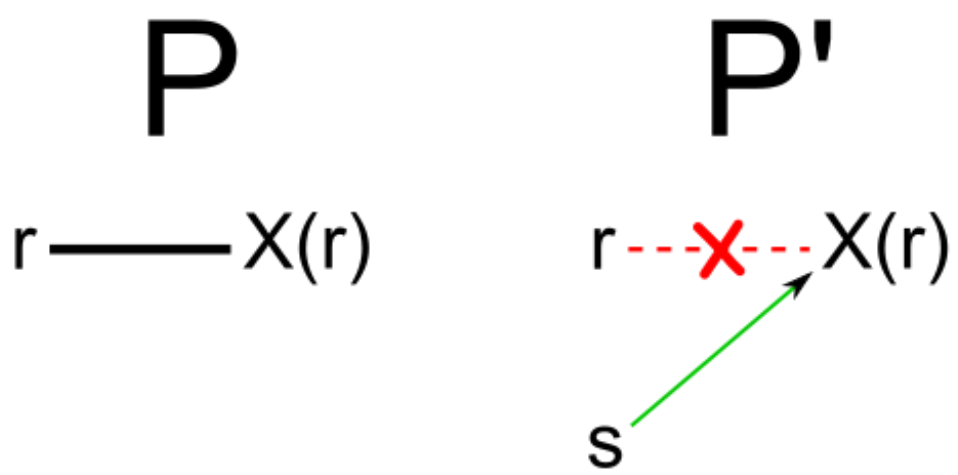

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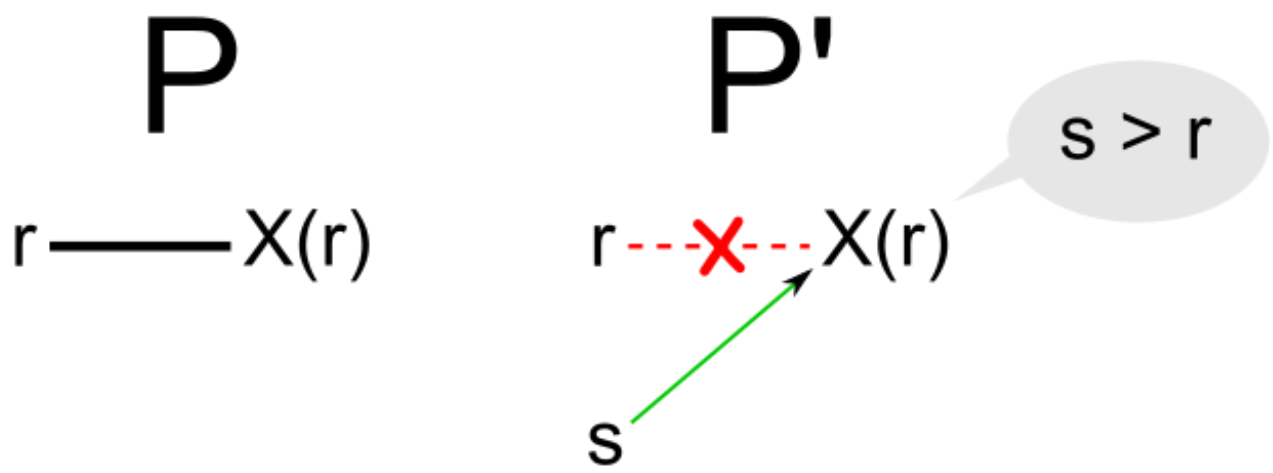

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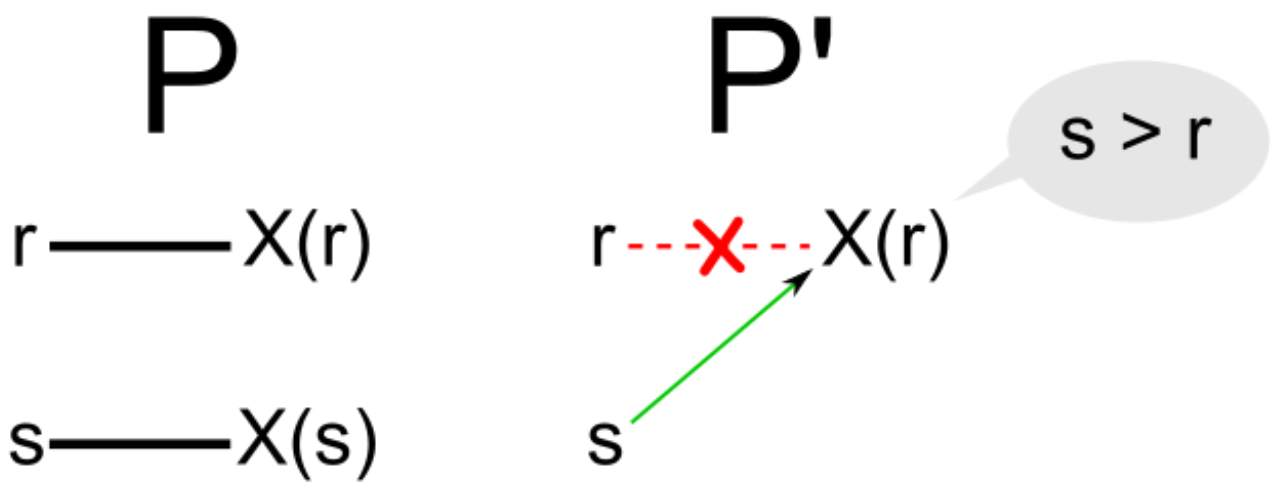

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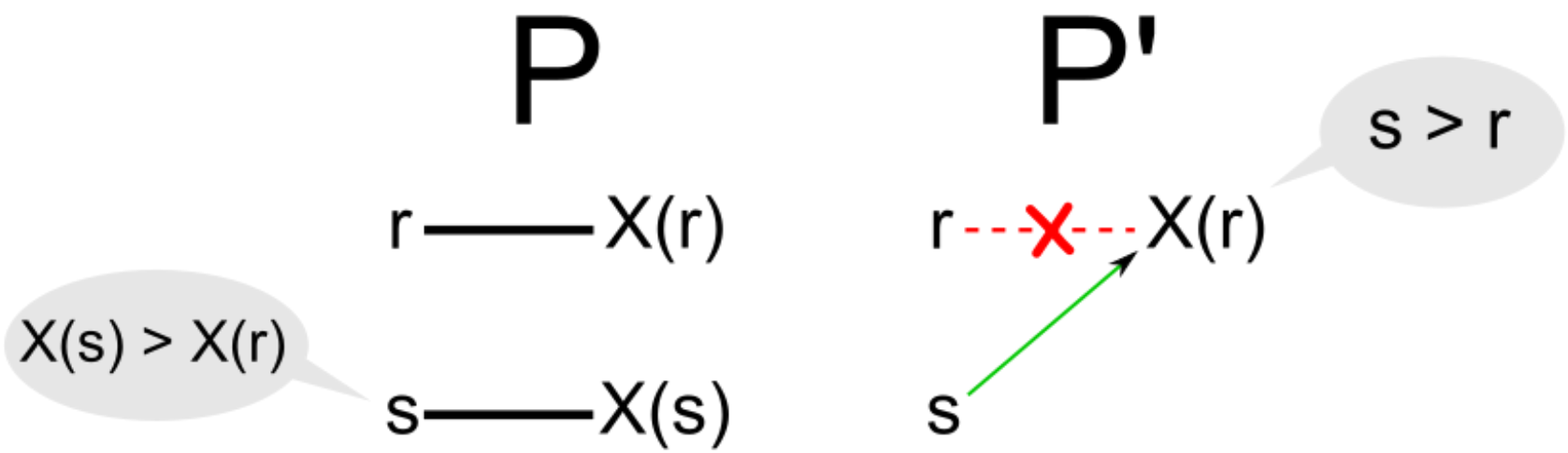

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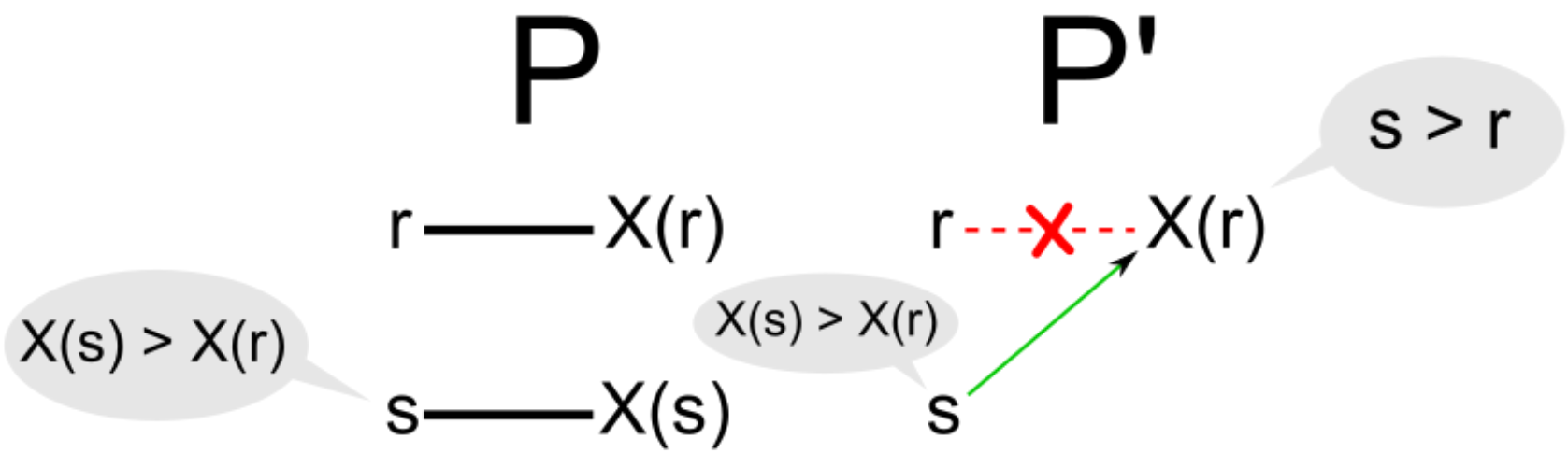

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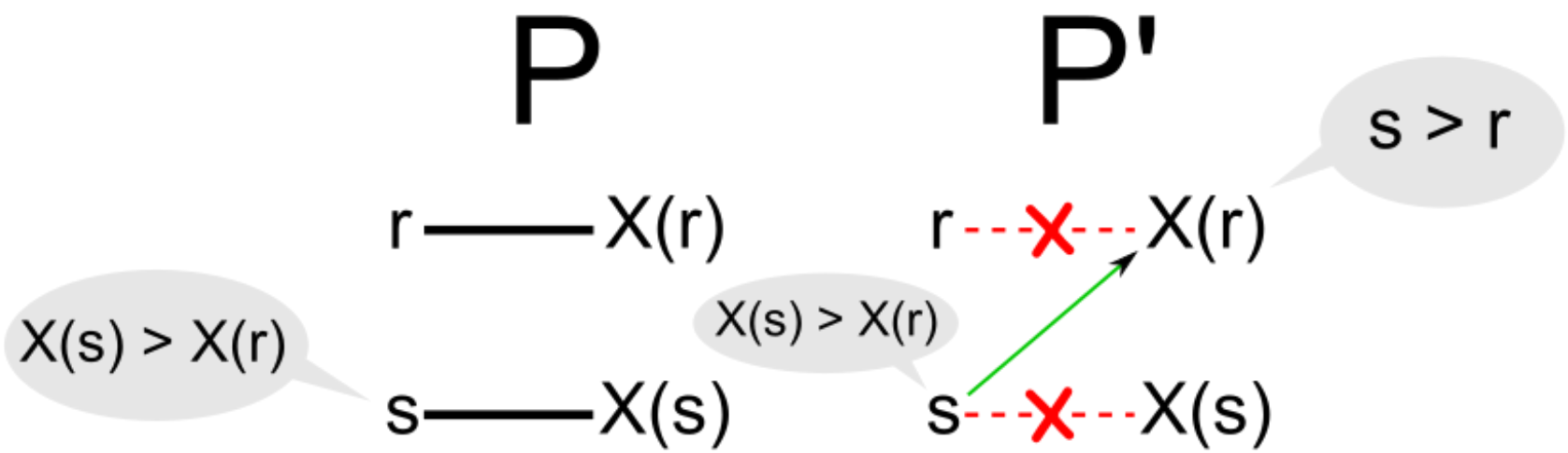

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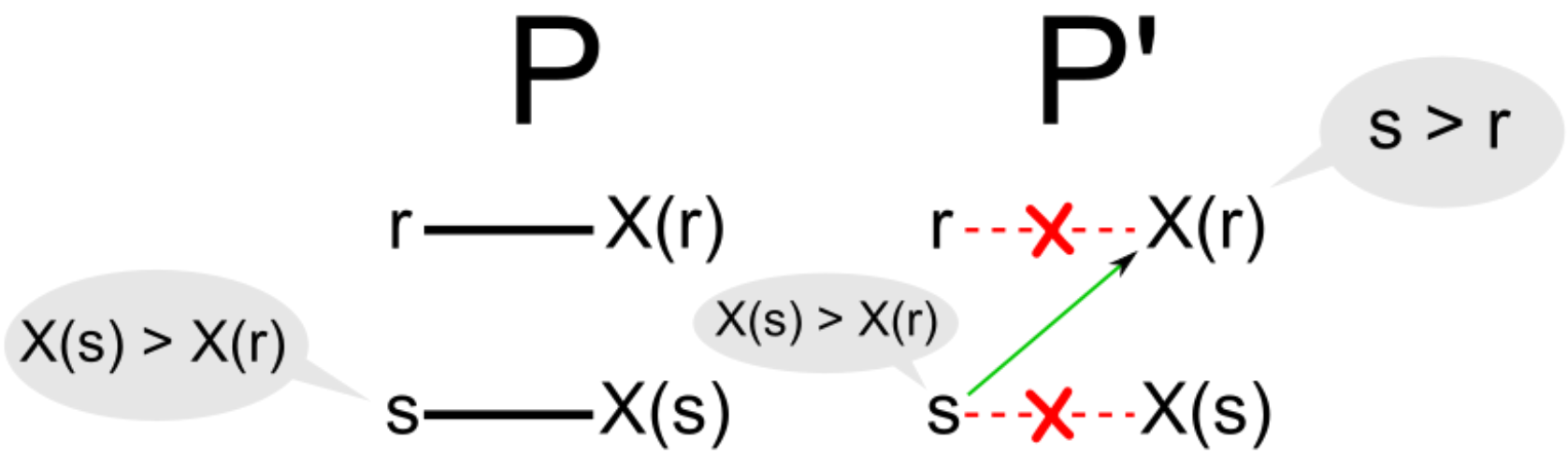

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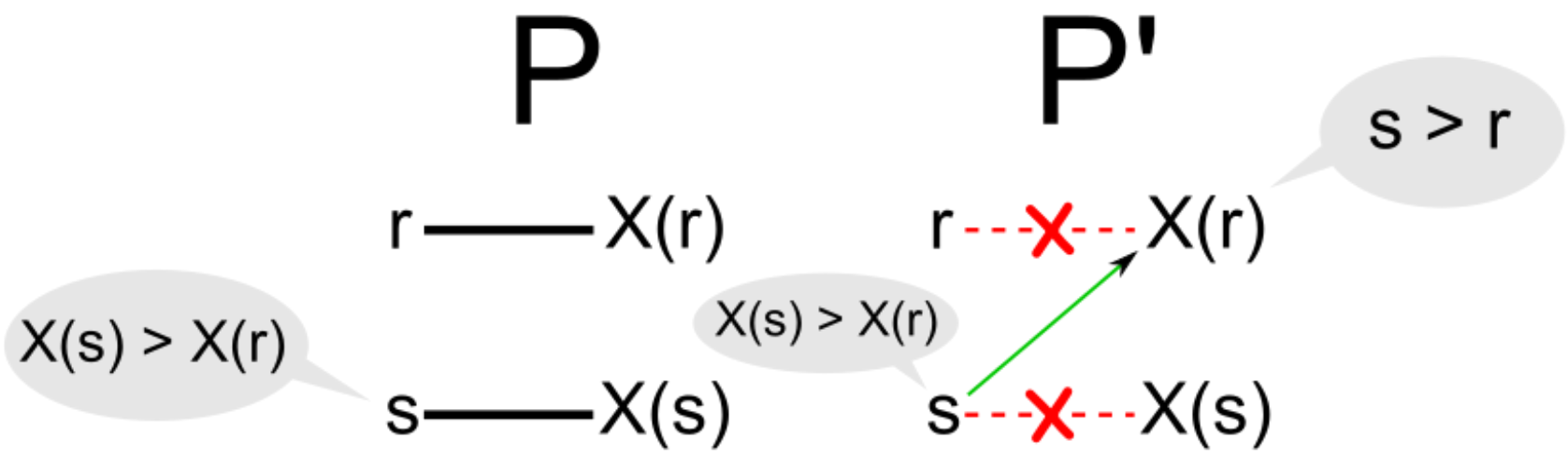

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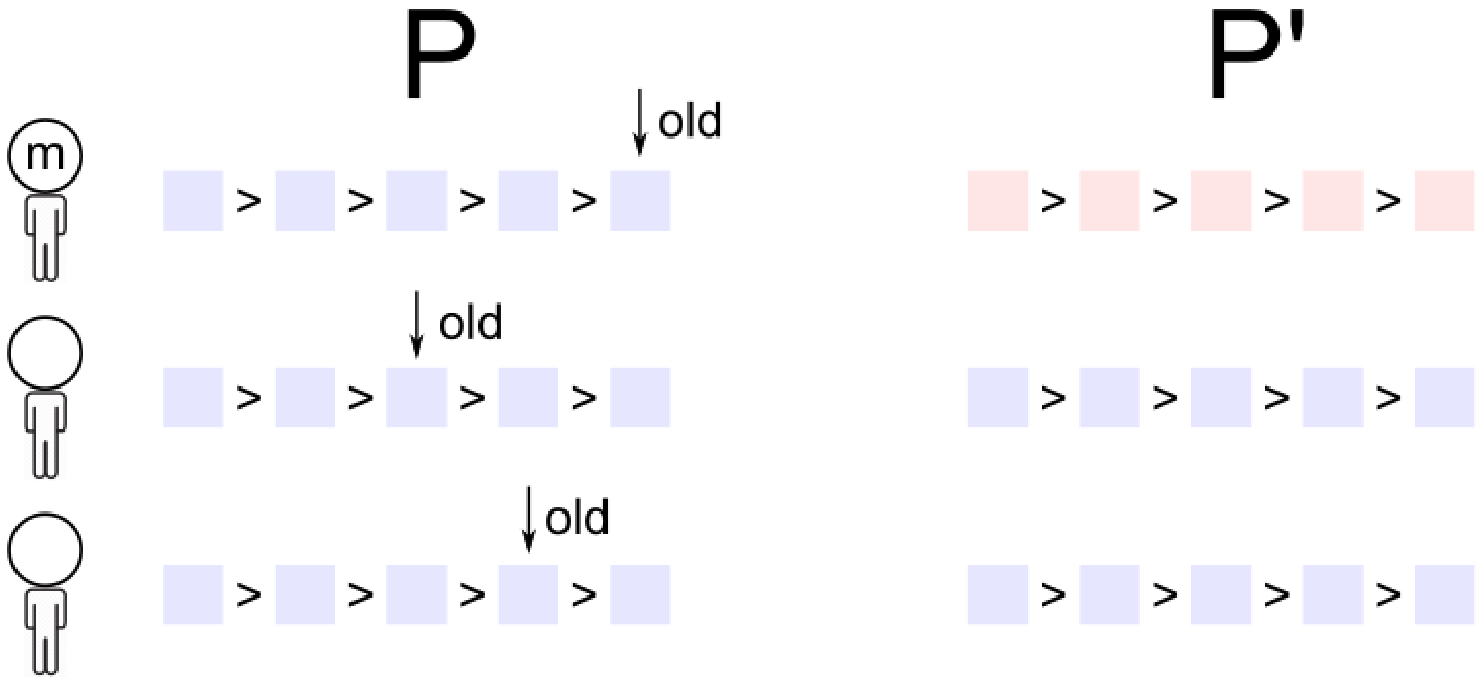

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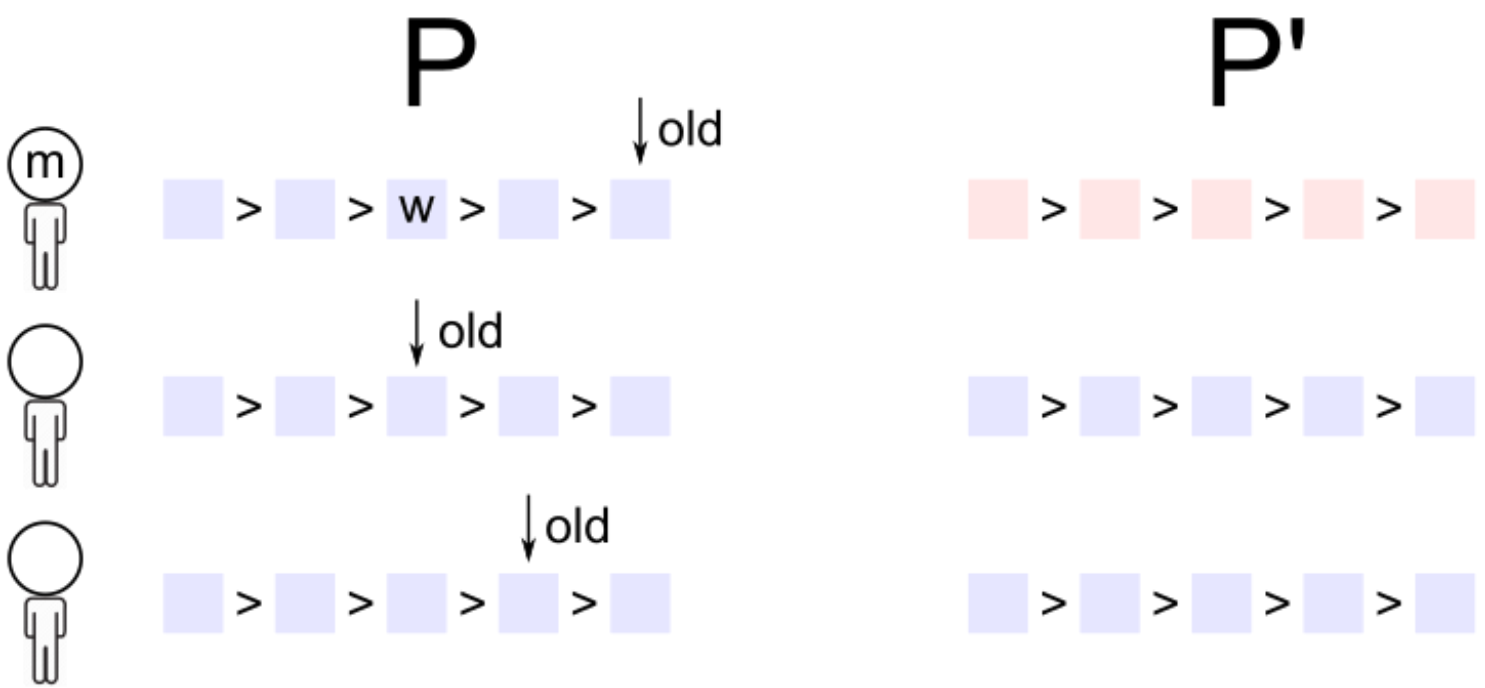

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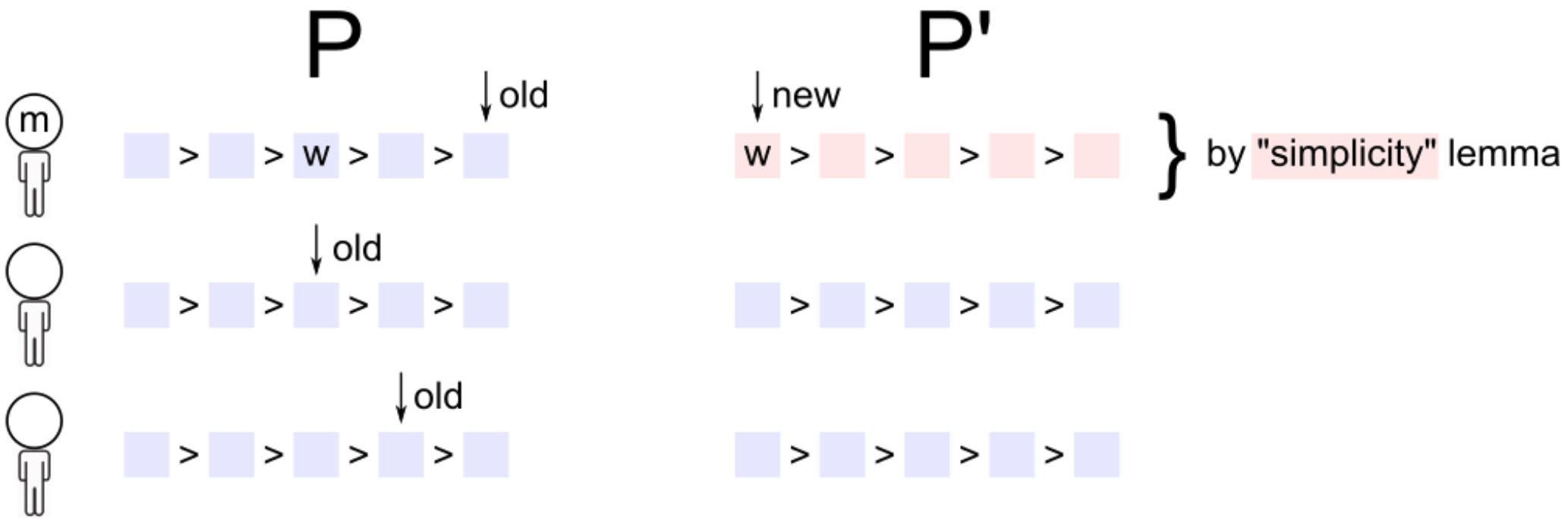

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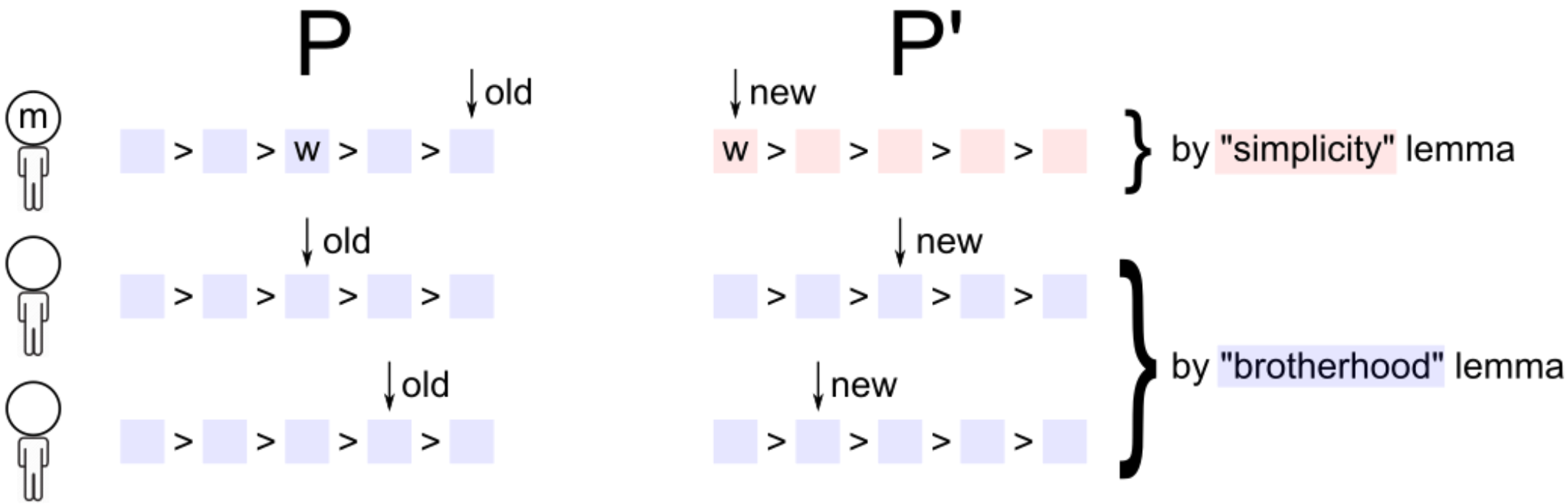

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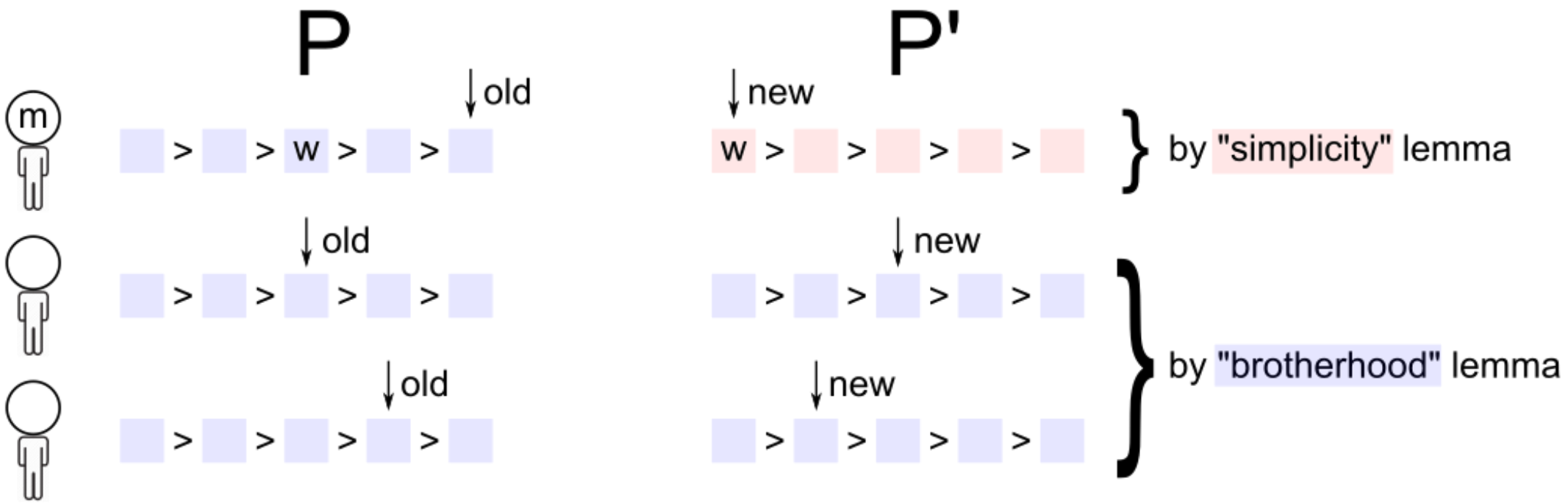

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If a woman receives just one proposal during DA on P,
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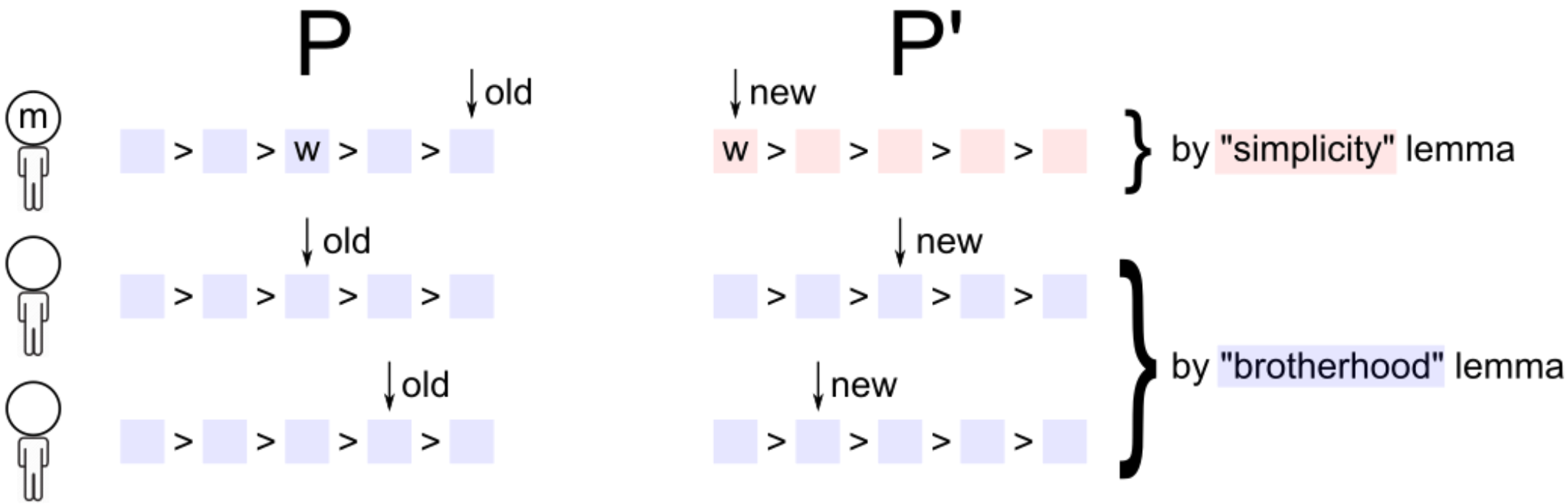

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Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

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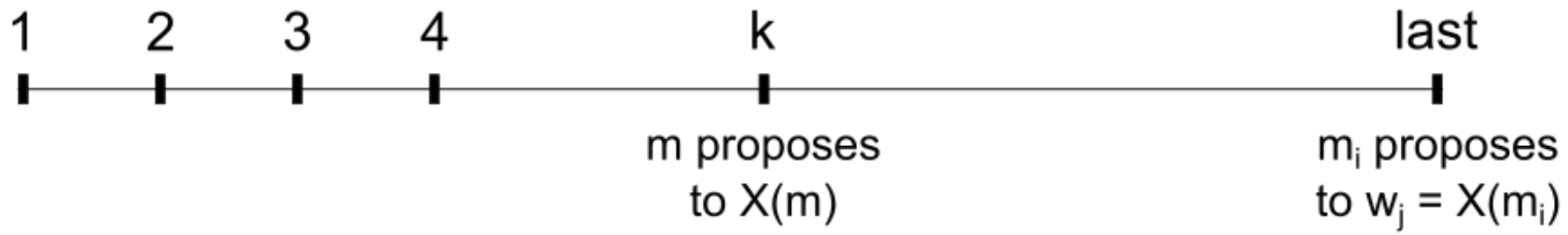

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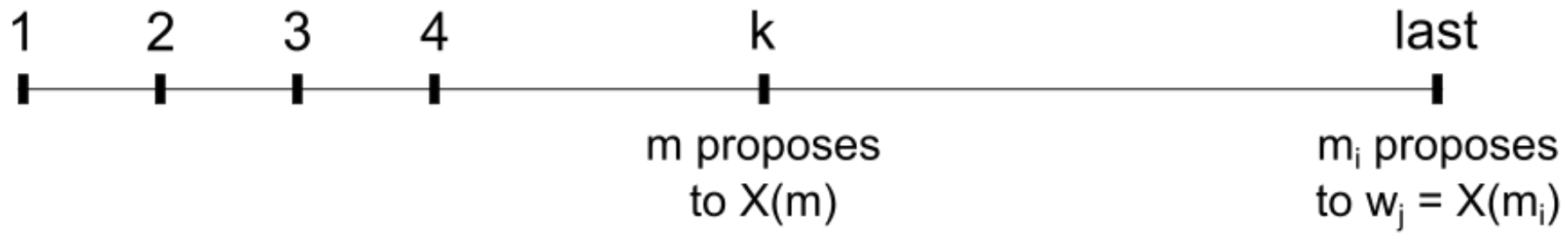

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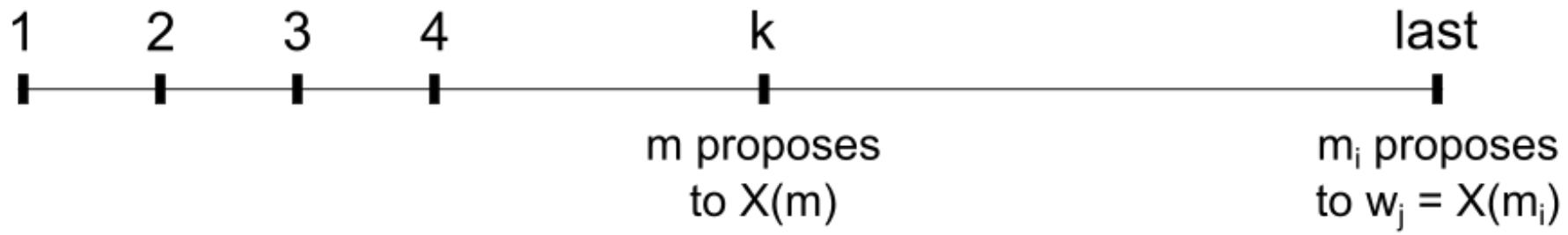

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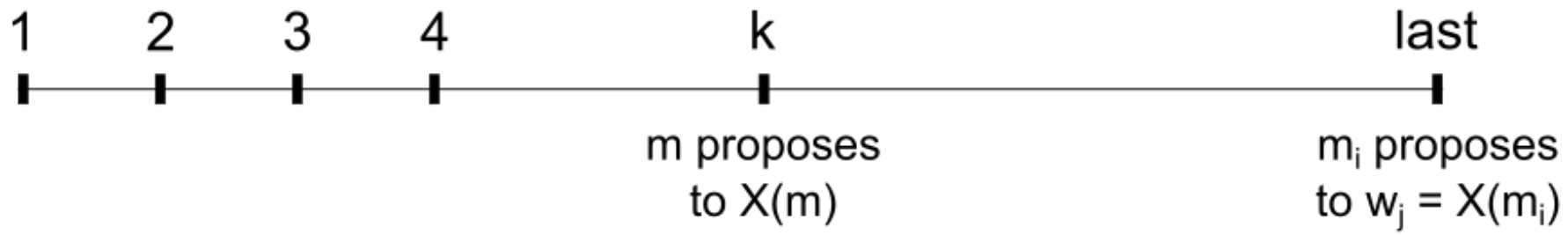

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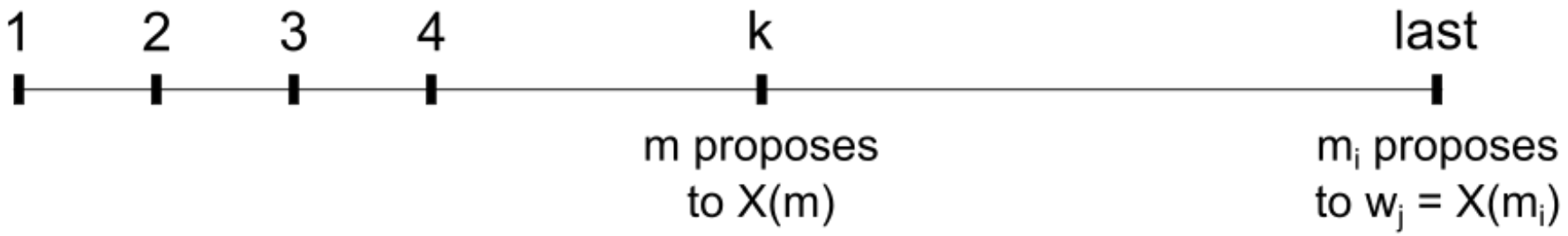

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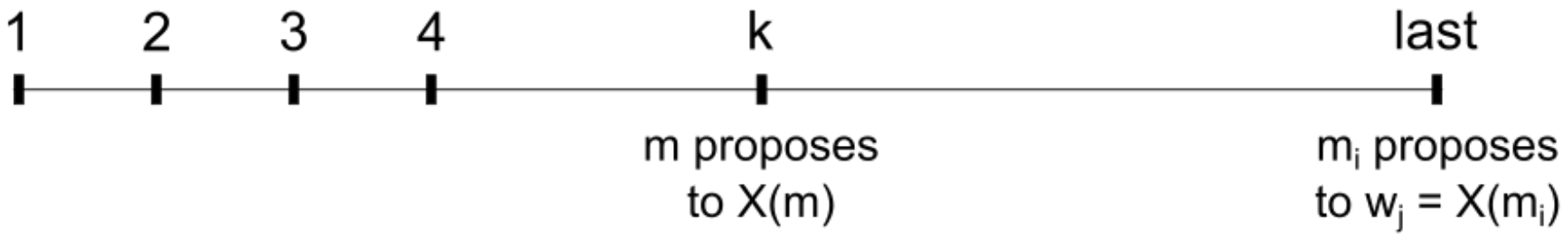

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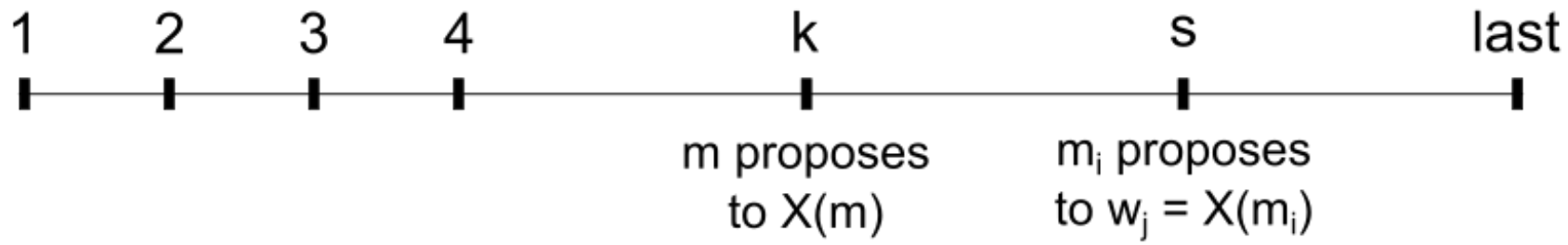

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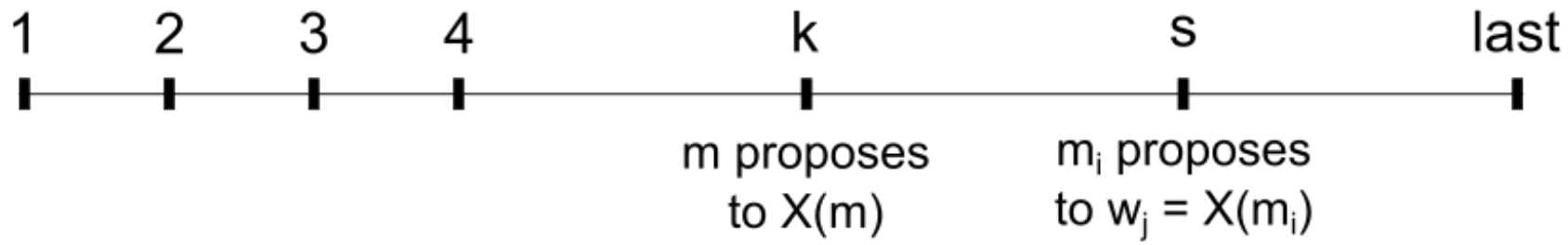

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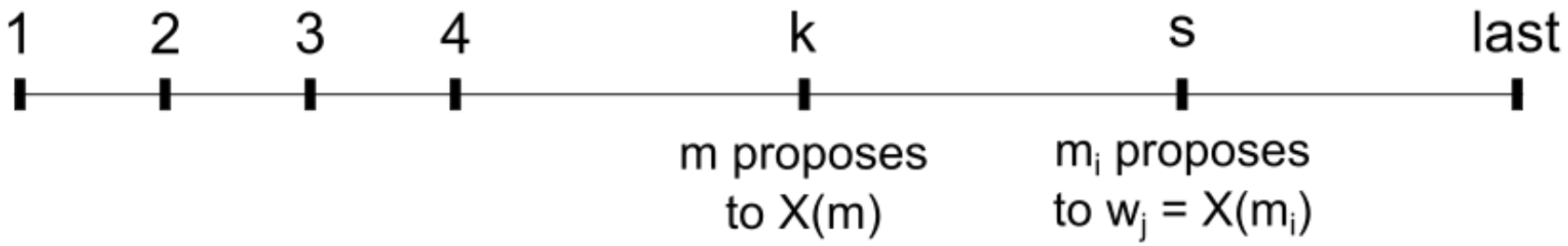
Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

Proof by induction.

Induction step:

Suppose the claim holds for all rounds s+1 or later, and we want to prove it for round s.



Suppose man m_i proposes to his X-partner $w_j = X(m_i)$ in round s of DA on profile P.

We want to show that $w_j = X'(m_i)$.

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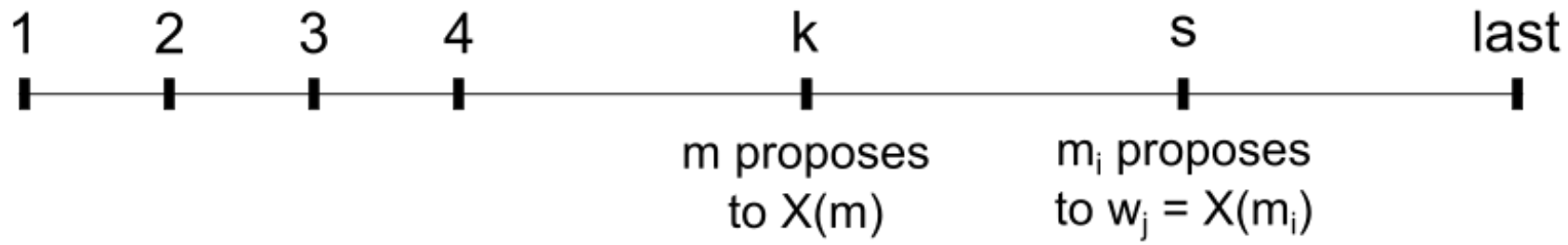

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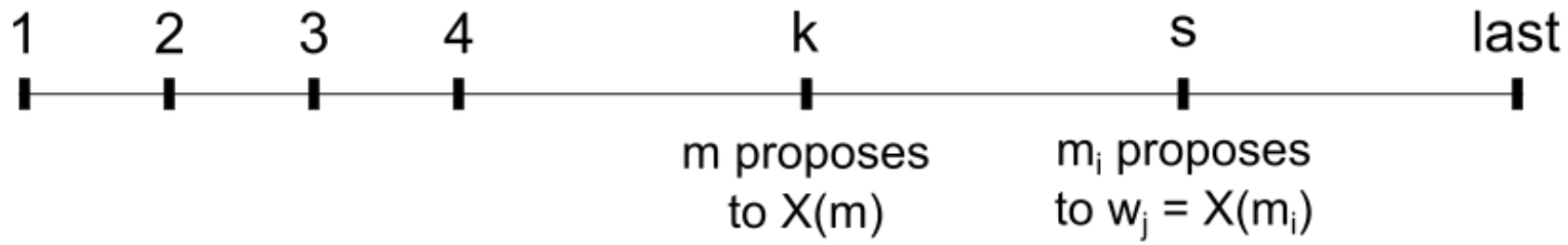

Let R = set of men rejected by w_j during DA on profile P (across ALL rounds).

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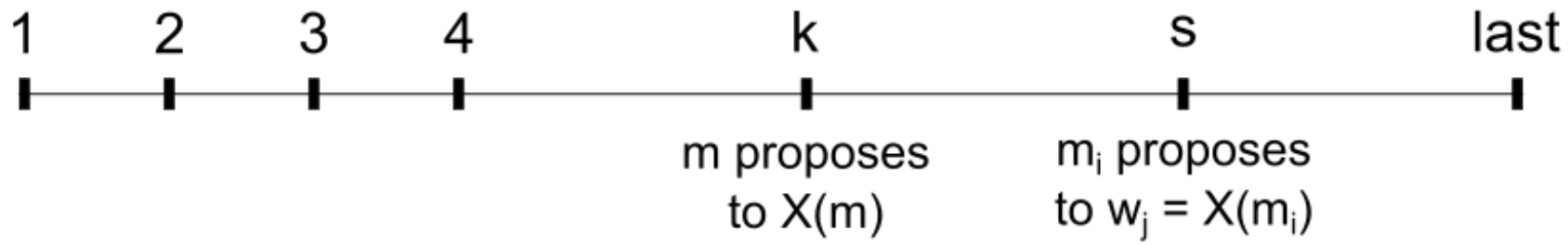

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If R is empty, then m_i is the only proposal that w_j receives.

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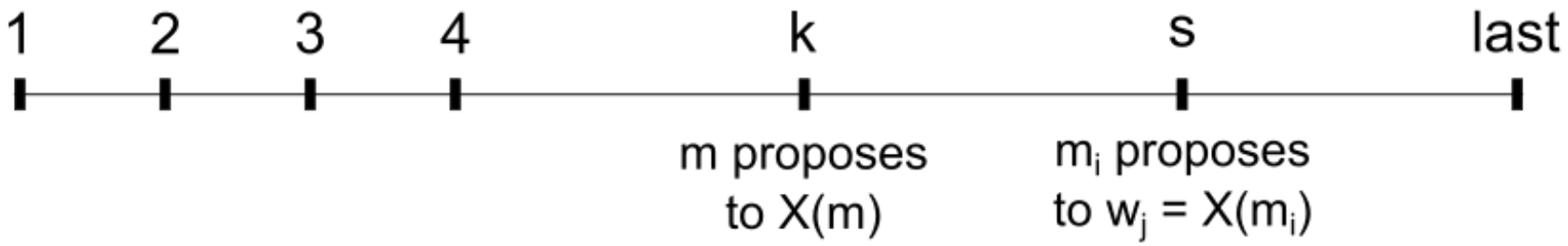

Let R = set of men rejected by w_j during DA on profile P (across ALL rounds).

If R is empty, then m_i is the only proposal that w_j receives.

By "no new proposal" lemma, $w_j = X'(m_i)$.

Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

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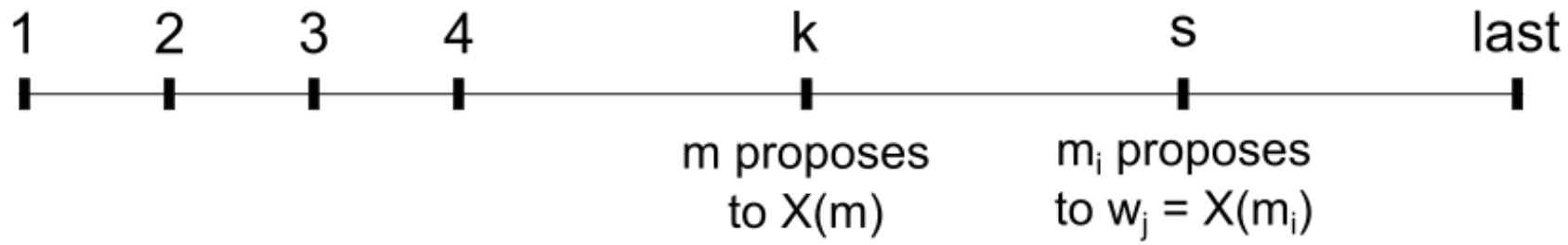

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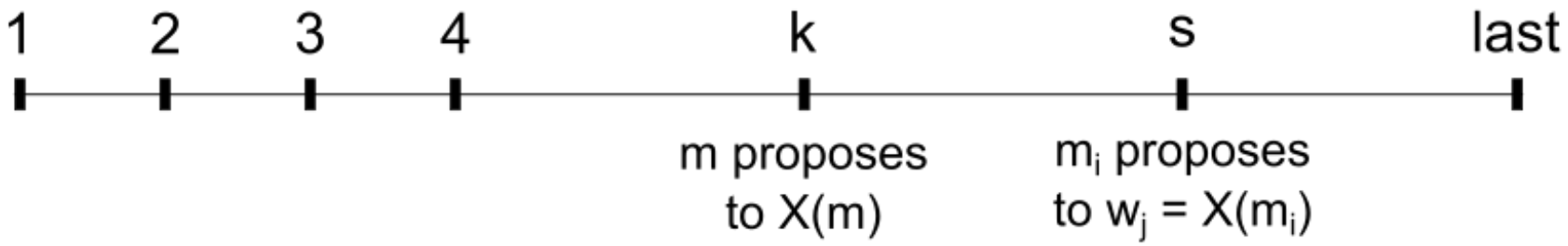

Let R = set of men rejected by w_j during DA on profile P (across ALL rounds).

If R is non-empty, then let m_F be w_j's favorite man in R.

Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

Proof by induction.

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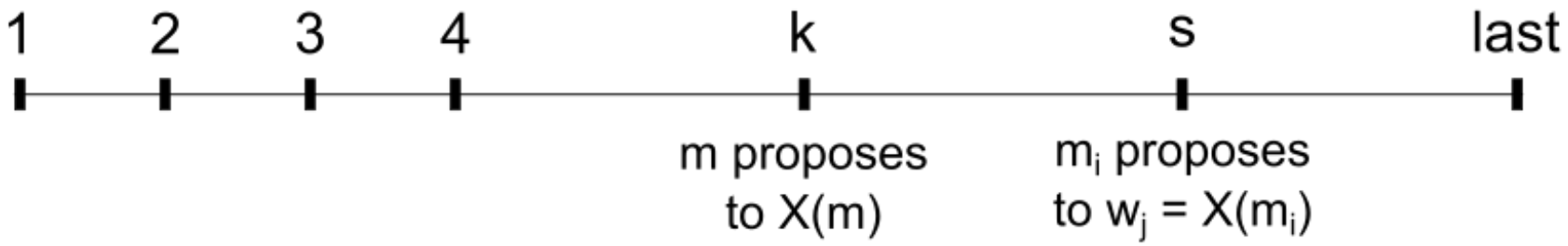

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Then, m_F proposes to his X-partner in round s+1 or later. Thus, $X(m_F) = X'(m_F)$.

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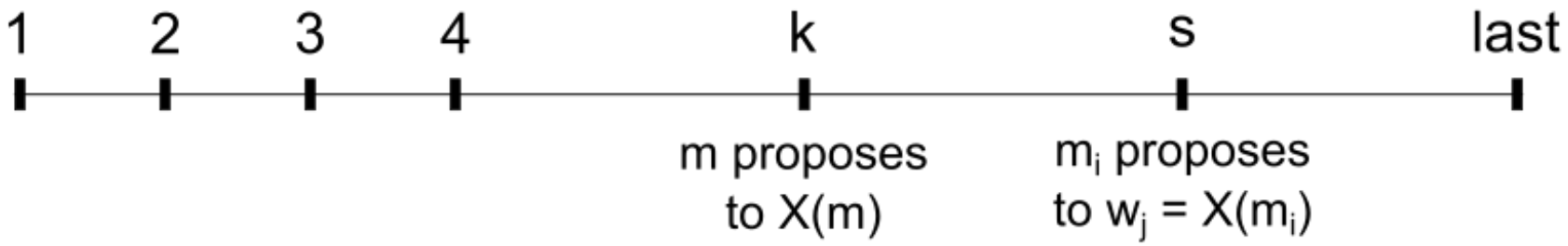

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This also means that m_F can't be the manipulator m.

Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

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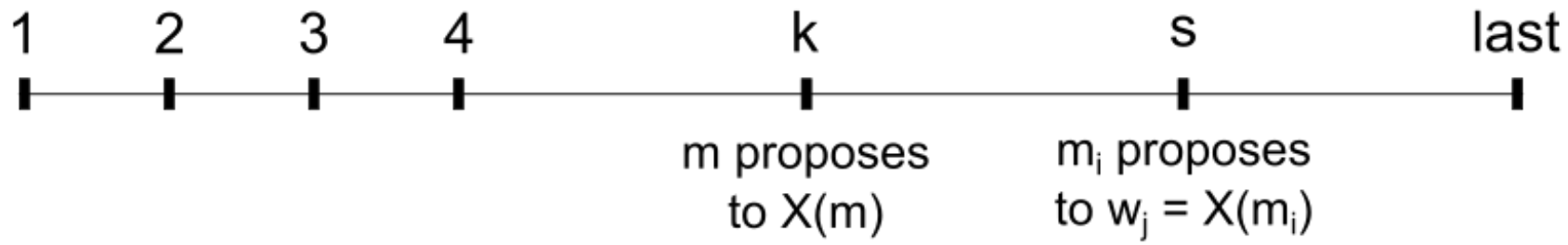

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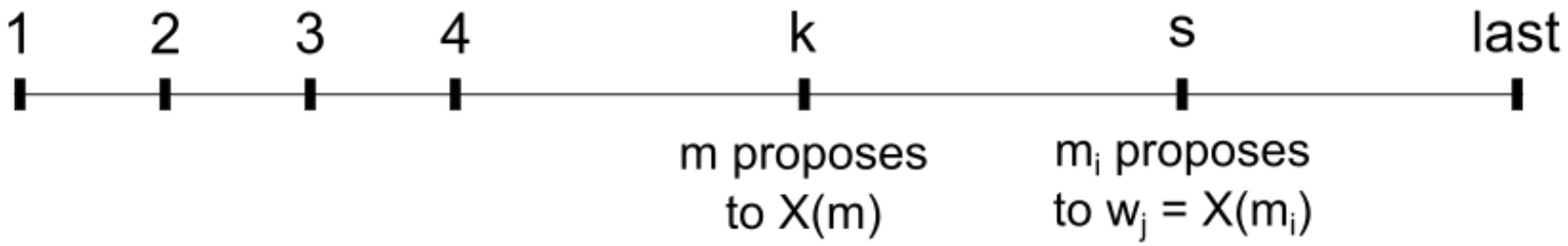

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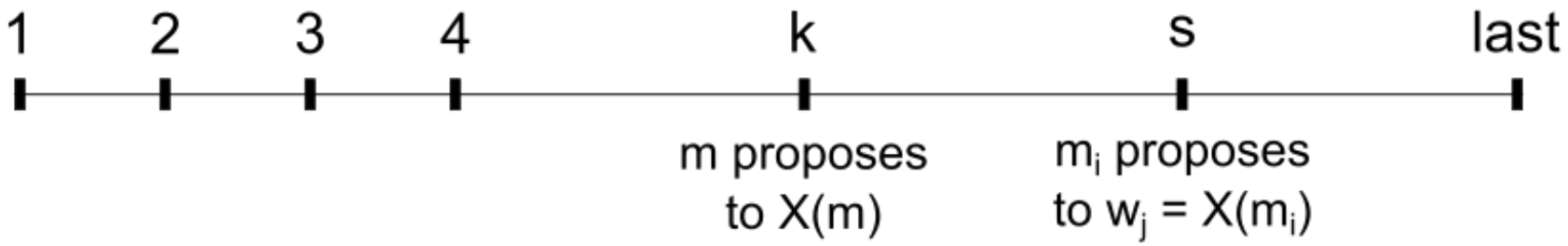

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So, m_F must propose to w_j during DA on P', and is again rejected since $X(m_F) = X'(m_F)$.

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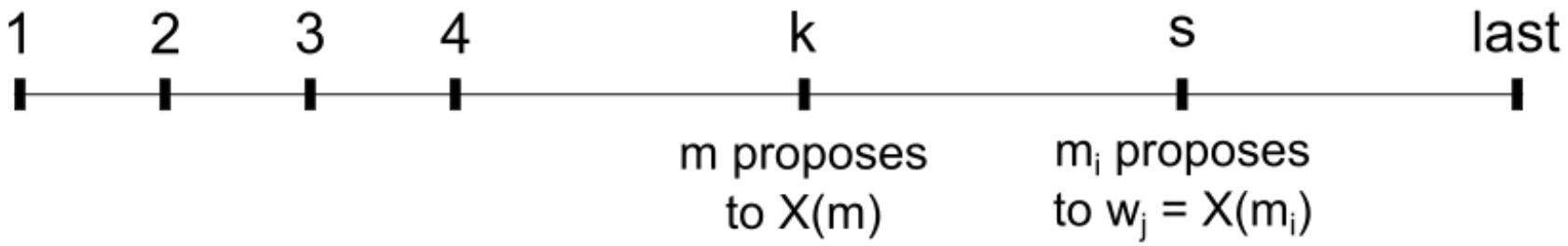

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Thus, w_j receives at least one more proposal (besides m_F) during DA(P').

Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

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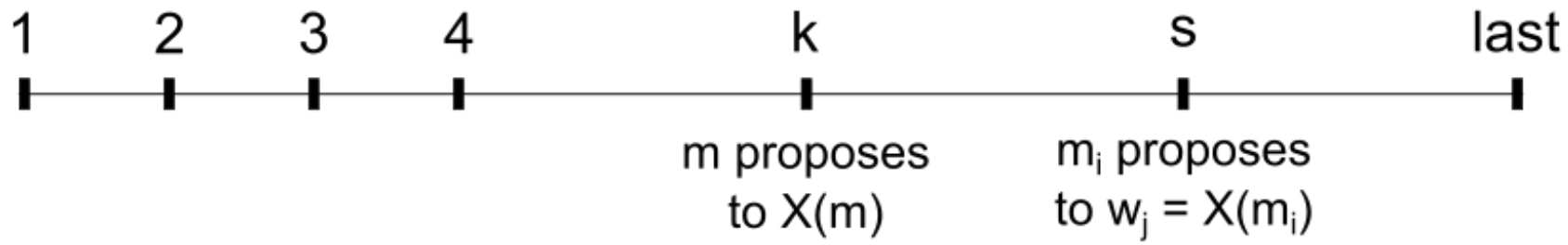

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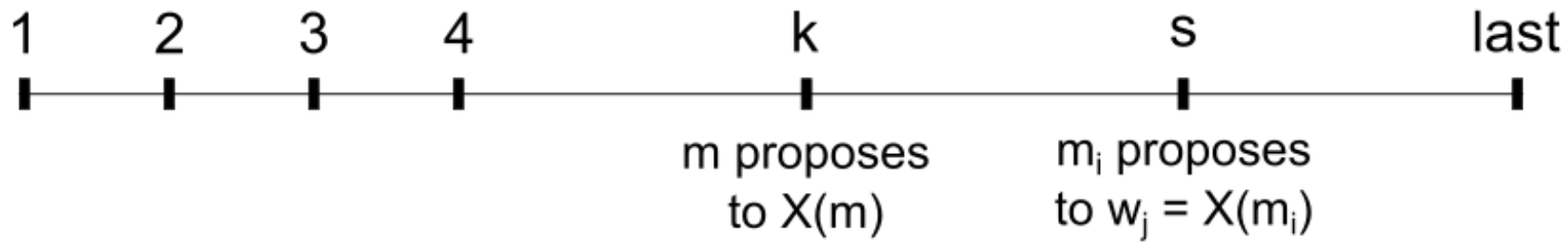

By "no new proposal" lemma, the man $m' = X'(w_j)$ proposes to w_j during DA(P).

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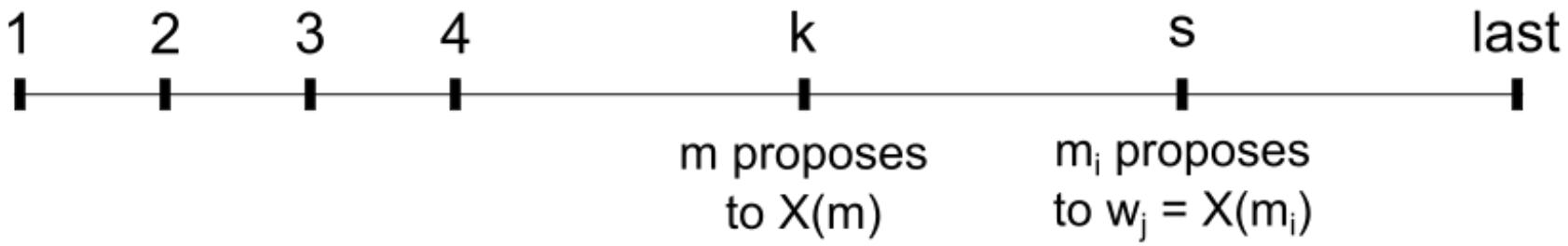

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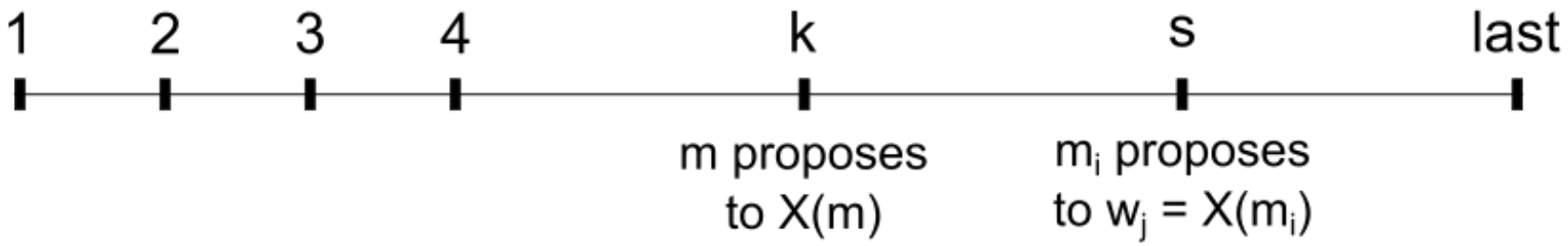

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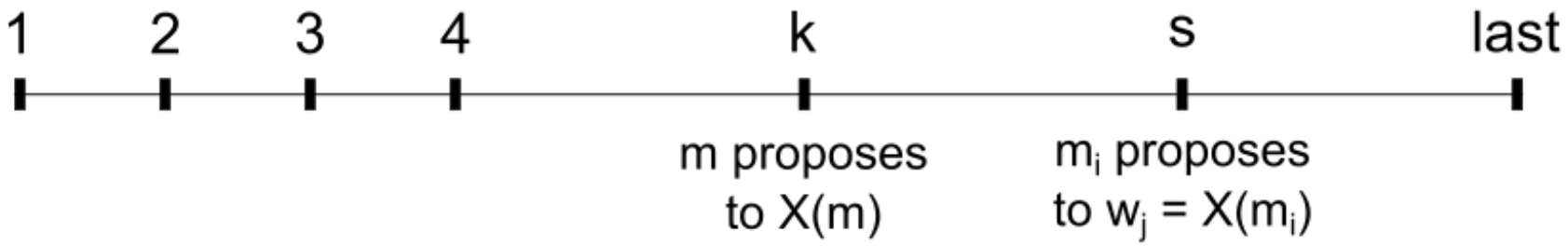

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- If $m_i >_w m'$, then w_j weakly prefers m_F over m', and would have rejected m' during DA(P'), again a contradiction.

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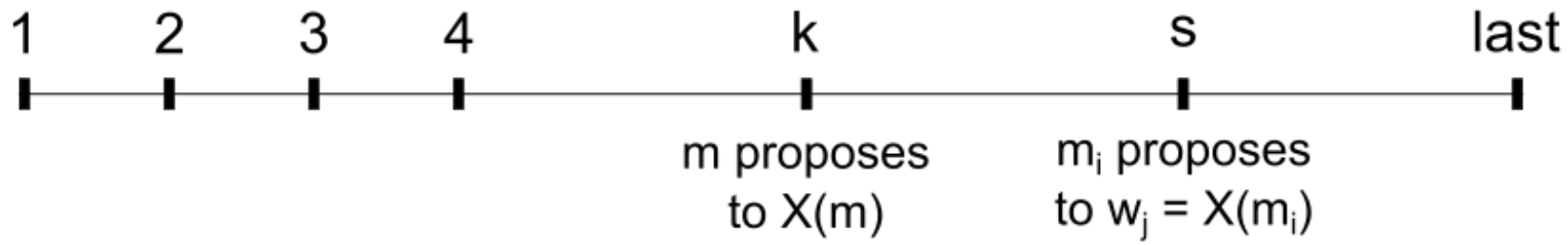

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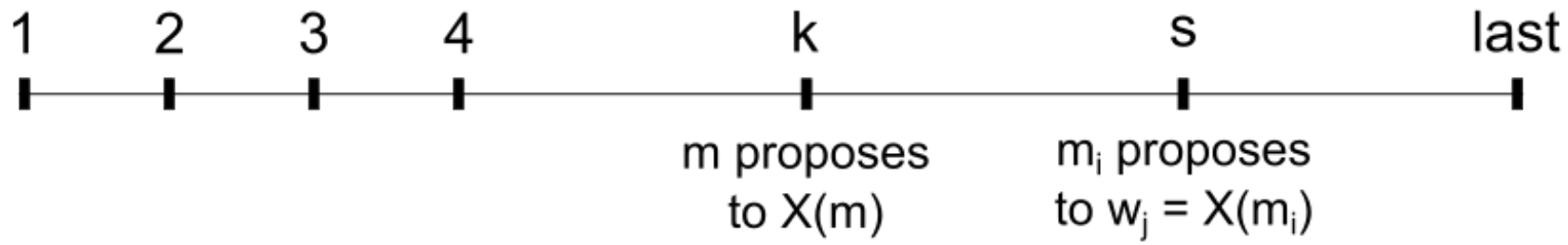

Therefore, $X(m_j) = w_j = X'(m_j)$.

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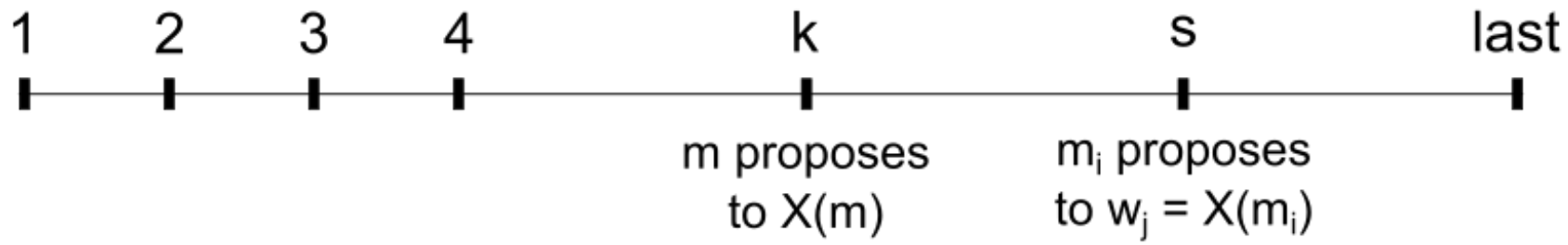

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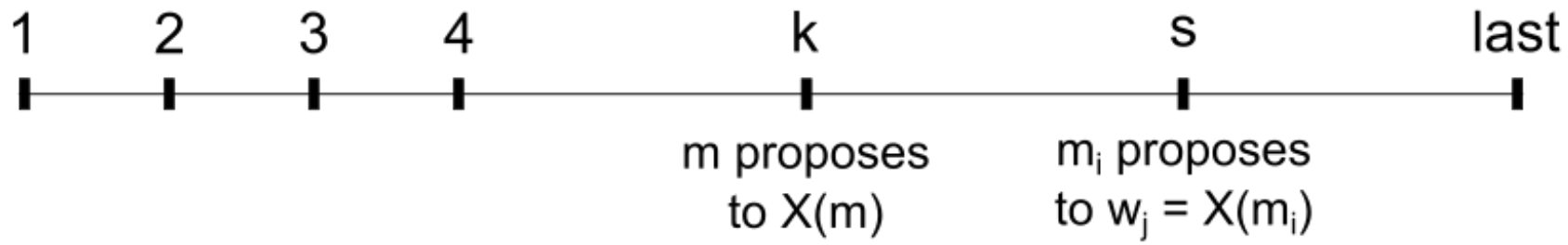

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[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is strategyproof for the men.

Suppose, for contradiction, that DA can be manipulated by a man m on the profile P.

Consider the execution of DA on the true profile P. Recall that X = DA(P).

Suppose man m proposes to his X-partner (i.e., X(m)) in round k of the algorithm.

Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

Then, X(m) = X'(m), which means the manipulator does not improve.

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